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Dating common commodity price and inflation shocks with alternative approaches

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Abstract. This paper investigates the occurrence of common price shocks (co-exceedance) across different commodities. IMF monthly price series of 11 commodities are considered over the 1980-2021 period. The analysis considers two alternative stochastic processes. The first looks for common volatility clusters using individual GARCH models to detect whether and when respective clusters overlap. Through an appropriate battery of tests, the second alternative looks for a common Bubble Generating Process (BGP) by searching for individual explosive roots and then dating them to identify the possible overlaps and first movers. Evidence emerging about these shock generating processes is linked to the analogous behaviour of the US Consumer Price Index (CPI) to assess to what extent inflation shocks can be associated to the observed commodity price spikes. Results show that the detection of temporary bubbles and volatility clusters only partially agrees on the episodes of exuberance, on the first-moving commodities and on the involvement of the CPI. This provides helpful suggestions on the development of a real-time surveillance tool supporting policy intervention in periods of commodity price turbulence.

Keywords: commodity prices, price volatility, explosive roots, GARCH models.
JEL Codes: Q11, C32.

1. INTRODUCTION

The large and rapid surge of most commodity prices that started in 2021 and lasted for the whole of 2022 points to two stylised facts that have been repeatedly investigated in previous episodes of price spikes: commodity prices move together; the rise of commodity prices transmits, somehow, to the Consumer Price Index (CPI). The consequent inflation rate rush largely impacts economies and societies and usually induces a quite vigorous policy response (Ider et al., 2023). Nonetheless, the explanations of these price dynamics are still to be fully understood.

The literature on the common movement (or co-movement) of commodity prices is vast (Byrne et al., 2020). One limit of this literature is that it implicitly assumes that the communality of price dynamics has to be intended as the existence of a common Data Generation Process (DGP), usually represented

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via some variant of Vector Auto-Regression (VAR) or Vector Error Correction (VEC) models or through more sophisticated representation of the underlying common drivers (for instance, common latent factors) (Esposti, 2021). But this may contrast with empirical evidence that suggests substantially different fundamentals across very diverse commodities, thus questioning the presence of common real determinants to justify commonality. In general terms, most representations of the common DGP and of the consequent price transmission process (like the conventional Granger causality, for instance) may be too simplistic to capture the real underlying interdependence across commodities, if any, thus providing misleading evidence on the actual causal linkages.

However, a specific strand of the recent empirical literature stresses that a common DGP is not strictly needed for a common temporary behaviour to be observed (Zhao et al., 2021, p. 781; Mutascu et al., 2022). In particular, commonality may only occur within the periods of exuberance, also referred to as *co-exceedance*. When the price spike expires each series reverts to its own (possibly different) normal-time DGP. This hypothesis can be also transferred to the second stylised fact, that of the CPI response: for a transmission of shocks to the CPI to occur we do not need a common DGP with the commodity prices, but only some co-exceedance with them.

The presents paper aims to contribute to this body of studies by proposing an original methodological approach which then leads to a novel policy tool. The main originality of the approach consists in juxtaposing and combining two alternative stochastic processes generating co-exceedance. The first resides in the occurrence of common (but not interdependent, that is, multivariate) volatility clusters whose behaviour is here modelled through appropriate Generalised Auto-Regressive Conditional Heteroskedasticity (GARCH) models. The second consists in the occurrence of common bubbles (a common Bubble Generating Process, BGP), that is, temporary explosive roots within the individual series but whose timing largely corresponds across commodities. Individual price series of very diverse commodities are thus separately investigated in order to assess whether and when volatility clusters (first) and temporary explosive roots (second) are found. Although these methodological approaches have already been adopted in previous empirical studies (Otero and Baum, 2021; Phillips and Shi, 2020; Zhao et al., 2021), this paper proposes a combination of these techniques to assess the co-exceedance of commodity prices without relying on some arbitrary and unreliable common DGP.

Monthly series of 11 commodity prices and the respective price indexes released by the International Monetary Fund (IMF) over the 1980-2021 period are considered. Co-exceedance is assessed by confronting the occurrence of these events across series. If some overlapping is observed, it supports the existence of some contagion (or transmission) across prices. The sequence of the events across prices can finally suggest the direction of this possible contagion. The same analysis is then repeated on the US CPI.

The interest for this methodological approach eventually lies in its application to design a suitable policy tool. Instead of concentrating on complex and possibly misleading causation processes, the proposed empirical strategy aims to identify when periods of rapid price rises occur and assesses whether they are common across commodities. Therefore, it allows to develop a real-time surveillance tool guiding a prompt policy response in the right direction, in particular by distinguishing interventions that can be confined to the sectoral context from interventions that require an economy-wide spectrum of actions. In order to be easily interpretable also by non-technical users, this tool is aimed to transfer results into a sort of periodically updatable dashboard visualizing the critical information under investigation: if a bubble is occurring for a given commodity, when it started, whether other commodities are involved by the same bubble, who moved first and, finally, if and to what extend this price surge is also reflected in the CPI. Contributing to the definition of such a policy tool represents a further objective of the present study.

The rest of the paper is structured as follows. Section 3 overviews the recent empirical literature in the field while Section 3 presents the adopted dataset and the main stylised facts. Section 4 details the adopted methodological approach, the results of which are illustrated in section 5. In Section 6 these results are discussed and juxtaposed with the evidence emerging from more conventional methodologies about the investigation of commodity price dynamics. Section 7 draws some policy implications and concludes.

2. THE COMMON MOVEMENT OF COMMODITY PRICES: LITERATURE AND EVIDENCE

The paper by Wang and Tomek (2007) may represent the first study that explicitly and extensively discussed the sequence of empirical issues to be tackled in investigating the actual DGP of commodity prices. Though their main attention was on the stationarity properties of agricultural commodities, their conclusions can be extended to other commodities and properties of the unknown DGP. The main argument is that, due to their market fundamentals (on both the supply and the demand side), commodity prices are expected to be mean reverting, with the long-term mean value possibly moving along a deterministic trend. So, prices are expected to follow a stationary DGP around a drift or a trend.

The fact that in the empirical literature the presence of a unit root is only occasionally rejected has to be attributed to the characteristics of the respective tests and/or to their misspecification. In particular, other characteristics of a stationary DGP can make it similar to a unit-root process. One is that these prices often show long memory (that is, fractional integration) making it possible for a close-to-(but-lower-than)-one root to be confounded with a unit root. Another is the presence of a structural break that may shift the long-term value upward or downward and can itself generate a potential confusion as evidence of nonstationarity: the presence of a structural break within a stationary series may lead to accepting the presence of a unit root, thus wrongly concluding that the series is non-stationary (Baum, 2005; Glynn et al., 2007).

A consistent body of recent studies concentrates on several different stochastic processes to explain the complex (i.e., non-linear) commodity price dynamics and the possible underlying co-movement. They are, in particular, fractional integration and structural breaks. A recent example, though concerning stock market indices and not commodity prices is Caporale et al. (2020). Based on an approach originally proposed by Cuestas and Gil-Alana (2016), they argue that fractional integration is very much related to non-linearities.[1] The possibility of structural breaks is also considered since many studies argue that fractional integration might be artificially generated by the presence of breaks in the data that have not been taken into account. In fact, the presence of structural breaks within commodity price series was already considered by Wang and Tomek (2007).

However, it must be noticed that fractional integration and/or structural breaks can hardly explain the behaviour of commodity prices and, in particular, the abovementioned co-exceedance, that is, their recurrent episodes of temporary exuberance as also emerging by simple visual inspection (see next section). They remain interesting and possibly relevant processes in the investigation of individual DGP since they may significantly interfere with the investigation of temporary bubbles and/or GARCH effects. Therefore, although the approach here adopted considers other DGPs, the presence of structural breaks can not be excluded at least for some of these commodities (Esposti, 2021) and will be considered here for comparative purposes (see Section 6).

Concentrating on the stationarity properties these studies overlook another major characteristic of these price series that clearly emerges from a simple visual inspection: the presence of temporary exuberance. Therefore, their DGP is expected to also generate self-extinguishing periods of particularly high or low values. Most of the literature in the last 15 years has essentially focused on this issue also as a consequence of the 2007-2008 price spike and of the following turbulent period. A lot of theoretical and empirical research has tried to investigate the origins of these price nonlinearities, jumps and spikes, as well as to put forward testing procedures to assess their presence. We can summarize this research effort in three main directions and, then, in their possible combination.

The first strand of research explains the observed price spikes and jumps as the consequence of a temporary increase in their variability (or volatility). It is the formation of volatility clusters that eventually generates the observed highly irregular price dynamics. In most applications, this idea is implemented by specifying and estimating GARCH regression models possibly admitting asymmetric effects and non-stationary processes for the price level. See Li et al. (2017), Baur and Dimpfl (2018) and Esposti (2021), just to mention a few, for the application of different variants of GARCH modelling to commodity prices.

Within the second body of studies the origin of the episodes of price turbulence is the formation of temporary bubbles. Several tests have been originally proposed to detect temporary price bubbles within mean-reverting, thus stationary, processes (Gürkaynak, 2008). More recently, the presence of temporary bubbles has been admitted, and tested, within possibly non-stationary processes, that is, as temporary explosive roots emerging within unit-root processes (Phillips et al., 2011, 2015; Phillips and Shi, 2020). Gharib et al. (2021) and Zhao et al. (2021) have recently used this battery of tests to assess the co-exceedance of some commodity prices and to date the respective bubbles.

Co-existence of both processes is also possible. This is considered helpful for two complementary reasons. On the one hand, as already anticipated, it is always difficult to clearly distinguish between the outcome of these two processes (Gürkaynak 2008, pp. 182-183; Chang, 2012). On the other hand, none of the two alternative processes may totally capture all the features of the observed price dynamics. To reconcile these two alternative processes, Chang (2012) adopts an Autoregressive Jump-Intensity(ARJI)-GARCH model. Originally pro-

[1] Another interesting strand of empirical literature on commodity price dynamics, and strongly linked to non-linearities and fractional integration, consists in the so-called fractal approach (Cromwell et al., 2000).

posed by Chan and Maheu (2002), this model is in principle able to generate both temporary bubbles and volatility clusters within stationary processes.

The third strand of empirical research in the field differs from the conventional time-series approaches as it is grounded on the spectral analysis and in time–frequency approaches. For the evolution of market prices, wavelet analysis has emerged as a useful and powerful tool in assessing price co-movement cycles. Without resorting on any theoretical causation (price transmission) process, it allows to explore how the series of prices are related at different frequencies admitting nonlinearities like structural breaks.[2] Mutascu et al. (2022) provide a valuable example of this kind of approach by investigating the co-movements of gasoline and diesel prices in different countries at different frequencies. Though this approach is relatively new, interesting and promising, it is still based on the assumption of a permanent interdependence between prices although flexible and not-linear. In the present study, as anticipated, we do not want to admit any persistent co-movement but only co-exceedance, therefore prices moving together only in specific periods of price spikes. Nonetheless, the combination of the co-exceedance analysis here proposed with wavelet analysis can open interesting developments for future research in this area.

Here, the aim is to investigate the commodity price dynamics following the first two relative recent strands of research by pointing to commodity price co-exceedance rather than co-movement. In particular, unlike Chang (2012) and Zhao et al. (2021) the objective is not to estimate the parameters of the actual DGP but to date the episodes of price turbulence by confronting, in this respect, two competing processes: GARCH within stationary processes (*volatility clusters*) and temporary explosive roots within non-stationary processes (*bubbles*). Moreover, unlike Zhao et al. (2021) here we do not adopt Granger causality testing to assess the direction of the possible transmission of the price shocks across commodities.[3] By dating these periods individually, we provide evidence on this transmission by solely juxtaposing the timing of the individual episodes.

This is done not only on commodity prices and price indexes but also on the CPI series. While the empirical literature on the commodity price properties and behaviour is vast and follows the abovementioned directions, the investigation of the CPI dynamics (and its growth rate, the inflation rate) mostly follows other directions. It mainly concentrates on the common movement and possible interdependence with other macroeconomic variables and is only occasionally connected to commodity prices (Garzón and Hierro, 2022; Ider et al., 2023). GARCH effects possibly occurring in the CPI or inflation rate series has been extensively analysed (Engle, 1982), but we are not aware of studies assessing the presence of temporary bubbles within these series. In fact, visual inspection seems to suggest quite different properties of CPI compared to commodity prices (see next section). Nonetheless, if a transmission from commodity prices to CPI is expected, especially in periods of price turbulence, this should imply some form of co-exceedance between these series.

But there is a final original aspect of the present contribution with respect to the recent literature in the field. It concerns the policy implications of the proposed empirical approach. In previous studies either these implications are overlooked or they concentrate on the possible effect of policy interventions on the nature and scope of commodity price co-movement or co-exceedance like, for instance, the fuel tax system (Mutascu et al., 2022) or import tariffs (Esposti and Listorti, 2018). If the main objective of a policy in this context is to minimize the negative impact of a generalized rise of commodity prices, knowing the possible underlying causation and transmission process, that is the structural linkages generating co-movement, might not be so critical. What seems important is rather a quick understanding that a price "bubble" is forming and whether or not it is just sectoral (so it involves a limited number of commodities) or it is generalized across all markets, that is, it is a co-exceedence. Sectoral interventions to neutralize a momentary price surge are present in many contexts and are usually rapidly activated (in the case of agricultural commodities, for instance, the agricultural market-crisis interventions represent an interesting example (FAO et al., 2011)). When occurring on first-moving prices, these prompt sectoral responses may help to prevent a generalized "bubble". Understanding if and when this latter is, in fact, occurring then becomes critical to promptly activate system-wide actions, particularly intended to prevent or slow-down downstream impact on inflation rate surges (Ider et al., 2023). This real-time surveillance tool able to provide such an early warning, as well as the generality and the first movers of the "bubble", seems to be particularly helpful for a prompt policy response.

[2] We wish to thank an anonymous reviewer for helpful suggestions on this aspect.

[3] Granger causality tests imply a common linear DGP across series (VAR or VEC models) (Zhao et al., 2021, p. 783). But both commonality and linearity may not hold in the present case. Nonetheless, for the sake of comparison and robustness check of results, in Section 6 we will present Granger causality tests.

3. PRICE SERIES UNDER SCRUTINY

The present analysis concerns the price of a selection of 11 commodities belonging to three different categories: 4 agriculture commodities (corn, wheat, soybean, beef); 3 energy commodities (crude oil, natural gas, coal); 4 metals (aluminium, copper, zinc; nickel).[4] All price series are taken from the IMF commodity price dataset.[5] All prices are monthly and cover the period January 1980 (1980M1)-December 2021(2021M12) (504 observations) with the only exception of natural gas whose series starts in 1985M1 (444 observations).

Together with individual commodity prices, the IMF dataset also contains aggregate price indexes for groups of commodities. Here, three monthly price indexes are considered: food price index (FoodInd) covering the period 1991M1-2021M12; metals price index (MetInd) covering the period 1980M1-2021M12; fuel (energy) index (EneInd) covering the period 1992M1-2021M12. Annex 1 provides details about which product quality these prices refer to, where they have been collected and on which aggregates respective indexes have been defined. Table A1 also reports the respective descriptive statistics which include the conventional distributional indices suggesting that commodity prices depart from the normal distribution mostly for a longer right tail depending on the exceptionally high prices observed during temporary bubbles.

The dynamics of commodity prices is investigated in combination with the evolution of the overall consumer price index (CPI). Unfortunately, no worldwide (or global) CPI is available. Moreover, many available CPI are usually collected and released at a quarterly or yearly basis. Here, the US monthly CPI series is used (see Annex 1 for more details).[6] This series seems suitable in the present analysis not only for the concordant frequency, but also because the US still represents the largest economy worldwide, so any impact of the global commodity prices on inflation can be consistently assessed on this series. It must also be noticed that, as detailed in Annex 1, several price series concern US markets and, in any case, all prices are expressed in US $. Therefore, using the US CPI does not incur the risk of downscaling (if not neutralizing) the transmission of commodity price shocks to the CPI due to the exchange rate adjustment (Garzón and Hierro, 2022).

Unlike many previous studies (Esposti, 2021), commodity prices, as well the three price indexes, are not deflated. As here we want to investigate the possible impact of commodity price spikes on the CPI, it does not seem appropriate to purge inflation from these series. The same strategy is followed for the possible presence of seasonality: no seasonal adjustment is performed on price series and indexes. The logic behind this choice is twofold. On the one hand, we prefer to analyse the price series that economic agents really confront with. On the other hand, as stressed by Wang and Tomek (2007) and Corradi and Swanson (2006), any data transformation has to be taken with care as it could introduce artefacts within the series under investigation.

However, we consider as appropriate a data transformation that is supported by the theory (Corradi and Swanson, 2006, p. 222). This is the case of the logarithmic transformation of the price levels. This transformation is largely used in empirical literature (Listorti and Esposti, 2012; Esposti and Listorti, 2013) and has two main motivations. First of all, price logarithms are more likely to show a normal distribution than price levels, and normality is usually required by the estimation and inference approaches. In other words, the log-normal statistical distribution of price levels has to be considered as a main regular feature of these series (Listorti and Esposti, 2012; Esposti and Listorti, 2013).

Secondly, the logarithmic transformation finds a robust theoretical justification in deriving the commodity price dynamics as Geometric Brownian Motions (GBM) (Diba and Grossman, 1988; Gürkaynak, 2008; Su et al., 2017). This tradition also includes the idea of "rational bubbles", that is, periods of price exuberance entirely justified by agent's expectations about commodity fundamentals (Diba and Grossman, 1988). Empirically, this hypothesis implies that price logarithms might take the form of mean reverting processes (due to market fundamentals) plus a random walk, a mean-reverting non-constant volatility (GARCH) and, possibly, temporary explosive roots.[7] According to Ibrahim et al. (2021), a GBM can generate a stochastic process that assumes normally distributed price level growth rates (therefore, difference in the logarithms) while admitting both unit-root (with drift and/or deterministic trend) and GARCH effects (volatility clusters).[8] However, these recent studies

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[4] Selected commodities are the most important worldwide (in terms of value) within the respective categories. In fact, nickel is the fifth in the list of metals after lead. But for this latter a sufficiently long series is not available.

[5] These price series are proprietary and can not be made available within the paper's material. However, they can be freely downloaded at https://data.imf.org/?sk=471DDDF8-D8A7-499A-81BA-5B332C01F8B9 or requested at https://www.imf.org/en/Research/commodity-prices.

[6] This data can be freely downloaded from https://fred.stlouisfed.org/series/CPIAUCSL.

[7] Actually, Diba and Grossman (1988) exclude that, within this logic, a rational bubble can actually start: if it is observed it must always have existed.

[8] See also Agustini et al. (2018) for a similar derivation.

do not admit temporary bubbles. Taking into account pros and cons of the logarithmic transformation (Corradi and Swanson, 2006; Wang and Tomek, 2007), the present paper considers both the price levels and the logarithm of price levels and in parallel repeats the analysis for these two cases in order to assess which results are robust across the transformation.

Annex 2 displays the time evolution of the three aggregate price indexes (Figure A1), the 11 individual commodity prices (Figures A2-A4) and the logarithms of these individual prices (Figures A5-A7) over the 1980M1-2021M12 period.[9] Visual inspection points to some general characteristics of the price dynamics. Within each group, commodity prices seem to show some common movements: periods of exuberance as well as collapses substantially correspond across different commodities. This is only partially confirmed across groups: metals and agricultural commodities tend to share the same periods of rise and fall, while energy commodity prices seem more stable and less volatile at least until the very last years of the period under consideration. However, if aggregate price indexes instead of individual series are considered, it emerges that the three series largely overlap with a substantial correspondence of positive and negative spikes. What is common across commodities is also that price turbulence seems to sharply increase in the second half of the period under consideration and, in particular, from 2005 onwards.

From this simple visual inspection, therefore, the hypothesis of common movement seems largely supported. For all commodities, periods of temporary exuberance are recurrently observed. During these periods, prices rapidly increase and then rapidly collapse to a level that does not differ much from the pre-exuberance level. Therefore, despite these "bubbles", prices still seem to behave like mean-reverting processes. This does not exclude changes in the long-term mean level or a long-term trend in this respect (Esposti, 2021). But these changes or trends seem mild and are overshadowed by the large short-term instability. As could be expected, the logarithmic transformation does not change the general behaviour of the series. Qualitatively, price levels and their logarithms are similar even though the latter are obviously smoother and this seems particularly evident for the energy commodity prices.

At the same time, major differences emerge between commodity price series and the CPI series. Figure A8 (Annex 2) reports the CPI, its monthly growth rate (i.e., the inflation rate) together with the oil price which arguably is one of its major drivers, but it is also one of the most stable commodity prices. The difference is evident. Oil price seems to follow a mean reverting process possibly with an increase of volatility in the second part of the period and an upward shift of the long-term mean value. CPI is much more stable, also in the second half of the period, and apparently moves along a deterministic trend. It follows that the inflation rate seems to behave like a mean-reverting process around an almost-zero long-term value with a limited, though appreciable, increase in the variability in the second half of the period.

This purely visual inspection gives rise to the two key research questions underlying the present study. On the one hand, commodity prices seem to move together at least during periods of turbulence, but this would suggest a common stochastic process whose properties, however, are not self-evident. Most price series show some characteristics of mean-reverting processes, and this would indicate they are stationary processes around drifts or trends. But the large and quick shocks, though temporary, do not seem consistent with this kind of processes. There should be some other underlying stochastic process, that may differ across prices but still admits their common movement at least in the periods of turbulence.

On the other hand, the research challenge about the linkage between commodity prices and the CPI is quite the opposite. They apparently behave as very different stochastic processes, so commonality should be excluded. Nonetheless, strong economic arguments, as well as an abundant empirical evidence (Garzón and Hierro, 2022), suggest that a common movement of many critical commodity prices has to be transferred, somehow, to the CPI.

4. THE METHODOLOGICAL APPROACH

The common theoretical framework of the investigation of commodity price dynamics consists in price formation mechanisms (or equations), that is, reduced-form models expressing the respective underlying market equilibrium.[10] Price formation equations represent the dynamic stochastic process as a mean-reverting or non-stationary process eventually generating the price level and volatility. These reduced form models have the further advantage of allowing a compact representation of cross-commodity price dynamics in the form of multiple

[9] The logarithmic transformation is not considered here for the price indexes and CPI. It would rather require a different aggregation of the elementary prices into the index and this would simply generate another kind of index possibly introducing a further artefact.

[10] Fackler and Goodwin (2001) provide a common template based on linear excess demand functions embracing all dynamic regression models from which an estimable reduced-form model can eventually be derived.

simultaneous equation models that may explain both co-movement and co-exceedance.

The theoretical justification of these cross-commodity price transmission mechanisms, however, is not univocal. The prevalent explanation is that also very different commodities (for instance oil and corn) may display interdependence in the respective fundamentals (i.e., demand and supply). For instance, on the supply side, one commodity (e.g., oil) may enter as an input (thus, a cost) in the production process or supply chain of another commodity (e.g., corn and, consequently, beef). On the demand side, consumption of one commodity may be directly (through substitution effect) or indirectly (through income effect) affected by the price of another commodity (Dawson et al., 2006; Listorti and Esposti, 2012; Esposti and Listorti, 2013). Sometimes, however, this interdependence through the fundamentals can be so indirect and remote that it seems more reasonable to provide another theoretical justification of price co-movement and co-exceedance: though prices are not interdependent, they still all respond to the same underlying (often latent) common factors (Stigler, 2011; Byrne et al., 2020; Esposti, 2021).

The research question underlying the present study, however, comes before these theoretical representations of price interdependence, that is nature and forms of price co-movement and co-exceedance. It rather looks for empirical support on the evidence of co-exceedance, its possible temporary nature and its dating. Therefore, we work on univariate models and not on multivariate models.

On these premises, consider N commodities whose price is observed over T time periods (months in the present case). On the basis of rational agent's expectation or efficient markets theory (Zhao et al., 2021), assume that for any i-th commodity there exists an unobserved fundamental price depending on the real market drivers (supply, demand, storage, expectations). The natural constraints applying to these drivers should make this market fundamental price nonexplosive. The actual (i.e. observed) price moves around this fundamental level but it usually deviates from it according to some underlying stochastic DGP expressed by the following univariate price formation equation:

$$p_{it} = \alpha_i + \delta_i t + b_i p_{it-1} + u_{it},\ \forall i \in N,\ \forall t \in T \quad S<T \tag{1}$$

where p_{it} is the i-th commodity price (or the logarithm of price) at time t; α_i expresses the drift while δ_i the deterministic trend coefficient. α_i, δ_i, b_i thus are commodity specific unknown parameters to be estimated. α_i and δ_i indicate the long-term fundamental price level or the long-term deterministic trend, respectively, to which the actual price is expected to revert.

The error term u_{it} is usually assumed to be normally, independently and identically distributed, that is $u_{it} \sim NID(0,\sigma^2)$. However, as autocorrelation in these disturbance terms is very likely to occur, (1) can be augmented to account for a transient dynamics:

$$\Delta p_{it} = \alpha_i + \delta_i t + \beta_i p_{it-1} + \sum_{s=1}^{S} \theta_{is} \Delta p_{it-s} + \varepsilon_{it},\ \forall i \in N,\ \forall t \in T \ \ S<T \tag{2}$$

where $\beta_i = (\beta_i\text{-}1)$ and θ_{is} are further commodity specific unknown parameters to be estimated. The error term is now correctly assumed to be $\varepsilon_{it} \sim NID(0,\sigma^2)$. (2) is the typical Adjusted Dickey-Fuller (ADF) regression and may admit different DGPs depending on the value of β_i. In particular, the price series is stationary, possibly around a drift (α_i) or a trend ($\beta_i t$), whenever $\beta_i < 0$. If $\beta_i = 0$, the price series contains a unit root and it thus follows a non-stationary process (a random walk) possibly with a drift (α_i) or a trend ($\beta_i t$). Finally, whenever $\beta_i > 0$, the price series has an explosive root implying a permanent and progressive departure from the fundamental price level unless it is temporary (a "bubble"). In practice, such process would contradict the actual existence of a fundamental price level.

Based on (2), distinct DGPs can be considered to represent the observed deviation of prices from the alleged fundamental level. Firstly, a Generalized Autoregressive Conditional Heteroskedasticity effect on ε_{it} can be included to capture the presence and persistence of volatility clusters. This is obtained by reformulating (2) as follows (GARCH(p,q)) regression model):

$$\begin{gathered}\Delta p_{it} = \alpha_i + \delta_i t + \beta_i p_{it-1} + \sum_{s=1}^{S} \theta_{is} \Delta p_{it-s} + \varepsilon_{it} \\ \sigma_{it}^2 = \gamma_i + \sum_{p=1}^{P} \rho_{ip} \varepsilon_{it-p}^2 \varepsilon_{it} + \sum_{q=1}^{Q} \omega_{iq} \sigma_{it-q}^2,\ \forall i \in N, \forall t \in T \quad S,P,Q<T\end{gathered} \tag{3}$$

where $\varepsilon_{it} = \sigma_{it} z_{it}$ with $z_{it} \sim NID(0,1)$. σ_{it}^2 is the it-h commodity price error term variance at time t, and ρ_{ip} and ω_{iq} are further commodity specific unknown parameters to be estimated. Together, parameters ρ_{ip} (also called ARCH terms) and ω_{iq} (called GARCH terms) express the overall degree of persistence of volatility. It is usually assumed that $\rho_{ip} + \omega_{iq} < 1$ (with $p=q$), indicating that volatility is mean reverting. Otherwise, we would be faced with a persistent volatility, i.e., volatility behaving as a random walk (or non-stationary) process (Engle, 1982; Agustini et al., 2018).[11] Once the GARCH model parameters have been estimated on the basis of the observed series, it is possible to assess whether and when volatility clusters occur. To do this, in-sample predictions of variance (i.e., $\hat{\sigma}^2_{1t}$) are generated. Then, on the basis of some pre-determined threshold (see below) clusters are found in those periods when this limit is exceeded.[12]

But a GARCH process is just one of the possible DGPs consistent with the observed irregular commodity price dynamics. As stressed by Engle (1982), a GARCH regression like (3) can be just an approximation to a more complex regression with non-ARCH disturbances. So, the GARCH specification might be picking up the effect of some relevant omissions from the estimated model. For this reason, we want here to make (3) compete with a second, and alternative, stochastic process generating a similar price behaviour. It consists of a DGP admitting temporary (or periodically collapsing) bubbles in the price levels. This DGP can be represented as a variant of the ADF regression (2) as follows:

$$\Delta p_{it} = \alpha_i^{r_1,r_2} + \delta_i t + \beta_i^{r_1,r_2} p_{it\text{-}1} + \sum_{s=1}^{S} \theta_{is}^{r_1,r_2} \Delta p_{it\text{-}s} + \varepsilon_{it},\ \forall i \in N,$$
$$\forall t \in Tr_1,\ r_2 \in T \quad S > T \tag{4}$$

where r_1 and r_2 denote the starting and ending points, respectively, of the possible temporary bubble. r_1 and r_2 are expressed as fractions of T so that $r_2 = r_1 + r_W$, where r_W is the window size of the regression, also expressed as a fraction of T. The number of observations to estimate (4) is $T_W = [Tr_W]$, where $[\cdot]$ is the floor function which gives the integer part of the argument (Otero and Baum, 2021). For series showing temporary bubbles we should observe explosive roots for some sub-periods, that is, some $[r_1,r_2]$ interval. This can be assessed through tests where the null hypothesis is $H_0{:}\beta_i^{r_1,r_2} = 0$, implying that the series shows a unit root, against the alternative hypothesis $H_0{:}\beta_i^{r_1,r_2} > 0$, implying that the series shows an explosive root in the $[r_1,r_2]$ interval.

A key contribution to a consistent formulation and implementation of this kind of tests was originally made by Phillips et al. (2011), then improved by Phillips et al. (2015) and Phillips and Shi (2020). The basic version of the test is the right-tailed ADF statistic based on the full range of observations, $r_1 = 0$ and $r_2 = 1$ (i.e., $r_W = 1$), denoted ADF_0^1. As it applies to the whole period of observations, this statistic may fail in detect short-time temporary bubbles. Therefore, a second statistic is based on the supremum t-statistic (SADF) that results from a forward recursive estimation of (4):

$$SADF(r_0) = sup_{r_2 \in [r_0,1]} ADF_0^{r_2} \tag{5}$$

Also this statistic may fail in the case of multiple temporary bubbles within the series. A third statistic can be thus computed. It is the generalised supremum ADF (GSADF) test:

$$GSADF(r_0) = sup_{r_2 \in [r_0,1], r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \tag{6}$$

Based on these statistics, it is firstly possible to asses if one or more temporary bubbles occur. Secondly, a backward testing procedure (backward SADF, or BSADF, statistics) allows dating these bubbles over the period T (Phillips et al., 2011; 2015). For any particular observation, i.e. the i-th commodity observed at time r_2, it is possible to test whether it belongs to a phase of explosive behaviour by performing a SADF test on a sample sequence where the endpoint is fixed at time r_2, and expands backwards to the starting point, r_1, which varies between 0 and $(r_2 - r_0)$. This backward SADF statistic is defined as:

$$BSADF_{r_2}(r_0) = sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \tag{7}$$

A further refinement of these tests has been recently proposed by Phillips and Shi (2020) and takes into account both the presence of heteroskedasticity and the multiplicity issue in recursive testing. They thus recommend a wild bootstrap approach to compute the critical values of the abovementioned tests.[13]

The methodological approach followed here can thus be summarised as follows. Firstly, we look for the stochastic properties of the individual commodity price series and the CPI. In particular, the presence of a unit-root (with or without a drift or a trend) and of ARCH effects is investigated. Secondly, on the basis of the first-step evidence, GARCH effects are considered as the possible explanation of the observed periods of price turbulence. GARCH regression models like (3) are estimated on individual series and in-sample volatility predictions are generated to assess and date the volatility clusters.

[11] This is also called Integrated GARCH (IGARCH) process/model (Campbell et al., 1996; Chan, 2010).

[12] Although their validity in generating reliable predictions is largely questioned, ARCH/GARCH models are usually quite successful in generating in-sample projections (Taleb, 2009).

[13] One limit of these tests is that they do not allow breaks in levels or time trends. As discussed, neither a trend nor a structural break can explain by itself the observed irregular price behaviour. However, they can not be excluded at least from some commodities (see Table 1) and might affect both the statistics and the critical values of these explosive root tests.

Thirdly, as an alternative to GARCH processes, we consider the formation of temporary bubbles as expressed by (4), therefore as a momentary departure from the fundamental process either stationary or non-stationary. ADF_0^1, SADF and GSADF tests are performed on individual series and the temporary bubbles, if any, are consequently dated by performing the BSADF test. Finally, the beginning and the end of volatility clusters and of temporary bubbles are confronted both across the two alternative processes and among commodities (and CPI) in order to assess similarities and differences, as well as the presence of possible contagion effects. Thus the analysis of co-exceedance simply consists in seeing whether volatility clusters or bubbles are common (i.e., overlap) or not. In case of a positive answer, it is then legitimate to ask, and to assess, whether a contagion effect can be deduced, that is, which series (i.e. price) moves first possibly driving the movement of the others.

Clearly, this investigation can not be confused with a formal causality assessment or testing. Usual time-series causality assessment in a multivariate context is performed via Granger causality testing. This latter, however, assumes a linear relationship across commodities and does not seem consistent with the observed stochastic properties of these series and bubble formation. In this respect, some recent developments in the field seem promising for future research (Shahzad et al., 2021; Esposti, 2022). It is worth stressing, however, that assessing causality is not so essential for the main policy implication of interest here. Investigating which commodities show a bubble formation earlier than others remains useful to build that real-time warning policy tool mentioned in previous sections.

5. RESULTS[14]

5.1. Stochastic properties of the series

Table 1 reports the battery of unit root tests and of the ARCH tests on (2) for all series under investigation. In the case of price indexes, CPI included, it emerges that all series are stationary. The selected specification[15] includes a drift in the case of the three commodity price indexes and a trend in the case of CPI. At the same, all indexes here show an ARCH effect except FoodInd. Consequently, all indexes behave as mean-reverting processes (with the mean moving along a deterministic trend in the case of CPI) possibly with volatility clustering.

Table 1. Unit root (ADF) and conditional heteroskedasticity (ARCH) tests on commodity price indexes and commodity prices (1980M1-2021M12)[a].

Series	ADF[b] (w/o drift&trend)	ADF (with drift)	ADF (with trend)	ARCH[c]
Price indexes				
FoodInd[d]	0.475	-1.224†	-2.439	0.126
MetInd	-0.088	-1.404†	-2.721	82.74*
EneInd[e]	0.150	-0.915†	-3.138	106.9*
CPI	6.988	0.539	-2.492†	72.91*
Price levels				
Oil	-0.502	-1.594†	-2.906	92.63*
Coal	-0.327	-1.599†	-3.415	160.9*
Gas[f]	0.581	-0.409†	-2.249	228.6*
Aluminium	-0.416	-3.063*	-4.010†*	65.84*
Copper	0.522	-0.621	-2.487†	112.8*
Zinc	-0.229	-2.023†*	-3.804*	106.9*
Nickel	-1.041	-2.628†*	-3.368	84.09*
Wheat	-0.375	-2.816†*	-3.349	108.9*
Corn	-0.214	-1.872†*	-2.891	29.86*
Soy	-0.322	-2.089†*	-3.288	67.44*
Beef	1.004	0.049	-1.739†	64.76*
Logarithm of price levels				
Oil	-1.199	-1.202†	-2.631	78.04*
Coal	0.429	-1.501†	-2.919	79.51*
Gas[f]	-1.737	-1.445†	-2.647	68.6*
Aluminium	0.178	-3.169†*	-4.322*	65.86*
Copper	0.784	-0.858†	-2.710	40.29*
Zinc	0.693	-1.827†*	-3.478*	19.18
Nickel	0.354	-2.008†*	-3.102	19.62
Wheat	0.252	-2.410†	-3.045	28.32*
Corn	0.315	-1.755†*	-2.855	7.38
Soy	0.212	-2.032†*	-3.243	24.67*
Beef	0.887	-0.167†	-1.839	42.35*

*Statistically significant at 5% confidence level.
† Selected specification according to Enders (1995, p. 256-260).
[a] The test specification in terms of lags included has been selected case by case on the basis of the AIC.
[b] 5% Critical Value of the three ADF test specifications, respectively: -1.95; -1.65; -3.42.
[c] Lagrange Multiplier (LM) test performed on the residuals of the ADF unit-root test equations; 5% Critical Value: 21.03.
[d] 1991M1-2021M12.
[e] 1992M1-2021M12.
[f] 1985M1-2021M12.

[14] All testing and estimation procedures have been performed with software STATA 17. In particular: GARCH models have been estimated using the command Arch with arch(1) garch(1) specification; explosive roots have been tested using the Radf command; the structural break tests have been performed using commands Zandrews and Clem; pairwise Granger causality tests have been performed by using, the Var and Vargranger commands.
[15] The best specification has been selected following Enders (1995, p. 256-260).

Regarding the individual commodity price series, however, a differentiated picture emerges across the commodity groups. If the levels are considered, energy commodities are all stationary around a drift. Metals, on the contrary, show non-stationarity around a drift in the case of zinc and nickel, non-stationarity around a trend in the case of aluminium and stationarity in the case of copper. Finally, all agricultural commodities, except for beef, are non-stationary around a drift while beef is stationary around a deterministic trend. Despite these difference, all commodity prices show an ARCH effect.

Interestingly enough, the logarithmic transformation changes the evidence emerging from the tests only for four commodities and only in one case (wheat) does this change concern stationarity properties. aluminium remains non-stationarity but now around a drift. Also copper and beef downscale from a trend to a drift while maintaining stationarity. Wheat shows the most significant change passing from non-stationarity around a drift to a stationarity around a drift. Thus, unlike the respective price level, the logarithm of the wheat price seems to behave like a mean-reverting process.

The key point, here, is that while visual inspection of both price indexes and price series would indicate some common movement, tests indicate that such commonality may occur for price indexes but not for individual prices where four different DGPs are observed, and this happens also within the same commodity group. This makes the hypothesis of common movement hardly tenable, at least over the whole time period. At the same time, however, visual inspection also reveals the presence of common periods of exuberance that are not necessarily compatible with the DGPs emerging from tests. The limited reliability of the DGPs emerging from the tests when compared to the actual price dynamics is confirmed by generating in-sample predictions from the estimated ADF regressions.

Figure 1 compares these predictions with the real series for two cases that should express different DGPs: a stationary series around a drift (mean reverting) (oil)

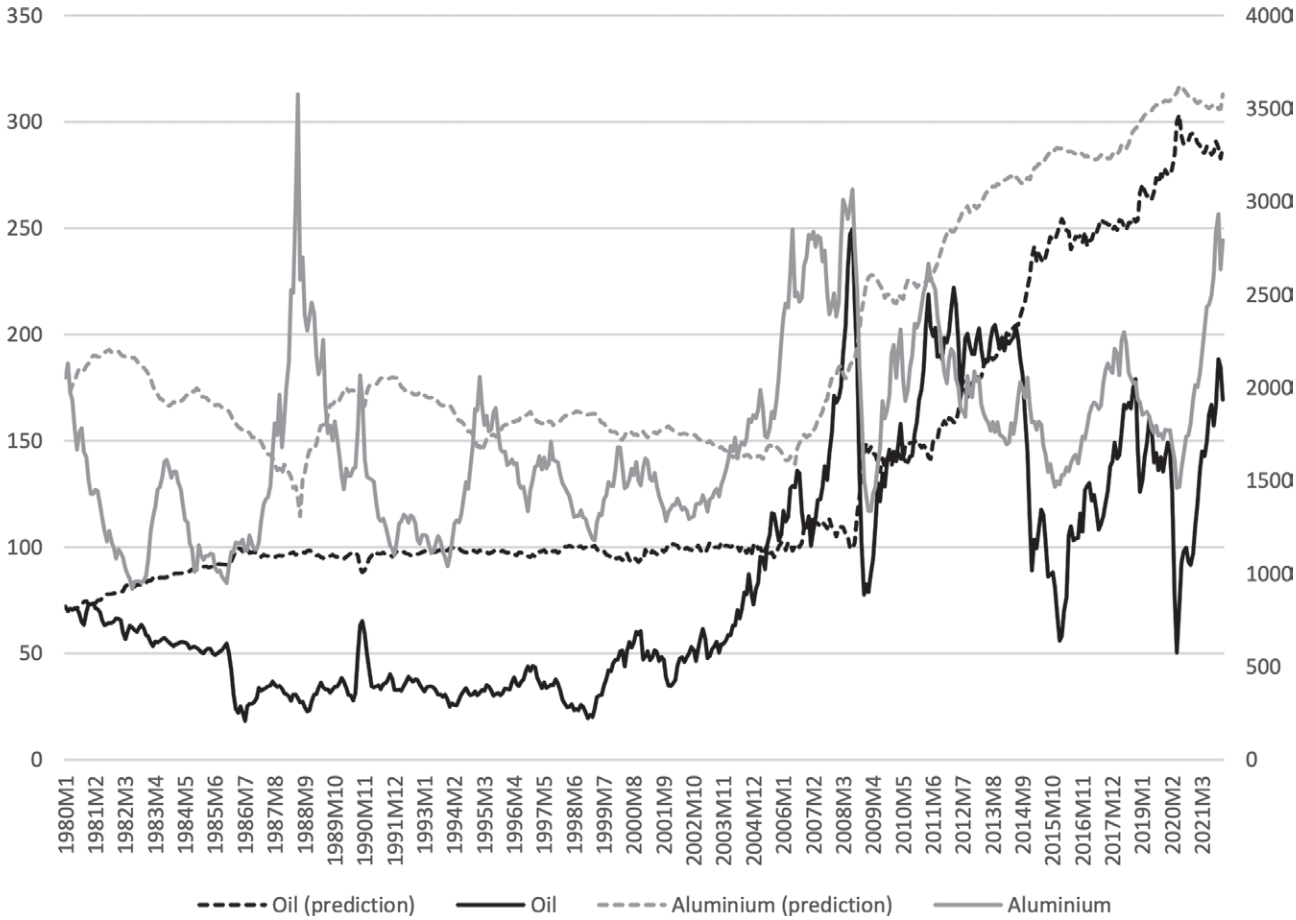


Figure 1. Oil (left scale) and Aluminium (right scale) prices: observed series and in-sample predicted series from respective ADF model estimation (1980M1-2021M12) (see Annex 1 for units of measure).

and a non-stationary series around a trend (aluminium). The two predicted series are quite similar, despite the different DGPs, and, above all, in both cases these predictions largely diverge from the actual series especially in the last third of the observed period. Evidently, there is something more in the stochastic process generating these series and this has to do more with temporary effects than with constant properties of the series. As the ARCH test is concordant across all series (except for FoodInd), the presence of volatility clusters can be a serious candidate to explain these temporary processes. But also temporary explosive roots (bubbles) could be considered as they are compatible with both stationary and non-stationary series over the whole period (Diba and Grossman 1988, p. 529).

5.2. Volatility clusters

Table 2 reports the estimates of parameters ρ and ω of the GARCH regression model (3) (with a GARCH(1,1) specification) for the different series in both price levels and logarithms. Two main facts emerge. First of all, with the only exception of FoodInd, in all series both estimated ρ and ω are statistically significant (Corn is the only case where ρ is not statistically different from 0). This confirms what was already obtained with ARCH tests presented in Table 2: volatility clusters occur in all series except for FoodInd. Secondly, many series violate the assumption of temporary clusters: for the price indexes EneInd and CPI, and price levels of natural gas, aluminium, zinc, wheat, corn, beef, we can not reject the hypothesis of $\rho+\omega$ =1. Therefore, in these cases volatility follows a non-stationary process thus making clusters permanent rather than temporary as expected. Logarithms of prices partially confirm this evidence but some differences are worth noticing: non-stationary volatility is observed also for oil and nickel while it is now excluded for aluminium, zinc and beef.

Contradictory evidence emerges about the reliability of these GARCH processes as generators of the observed price dynamics. On the one hand, the existence of volatility clusters is consistent with the observed large variability, or instability, of the commodity prices in specific periods of time. On the other hand, however, in several cases these processes support permanent volatility shocks thus becoming less compatible with the observed temporary episodes of turbulence. As discussed, once estimated, standard error in-sample predictions for these GARCH models can be generated. Figure 2 shows these predictions for the three price indexes and Figures 3a-3b for the individual price levels and logarithms, respectively. Figure A9 (panel a)) reports the same predictions for the CPI and its growth rate (i.e., inflation rate).

Table 2. GARCH(1,1) model estimation and persistency test on commodity price indexes and commodity prices (1980M1-2021M12) (estimated standard errors in parenthesis)[a].

Series	ρ		Ω		Test $\rho+\omega$ =1 ($\chi^2(1)$)
Price indexes					
FoodInd[b]	0.036	(0.049)	-0.171	(0.717)	3.96*
MetInd	0.247	(0.049)*	0.675	(0.032)*	4.24*
EneInd[c]	0.373	(0.081)*	0.641	(0.058)*	0.16
CPI	0.116	(0.022)*	0.879	(0.021)*	0.22
Price levels					
Oil	0.343	(0.041)*	0.726	(0.031)*	10.4*
Coal	0.494	(0.062)*	0.664	(0.026)*	13.4*
Gas[d]	0.304	(0.051)*	0.625	(0.044)*	3.52
Aluminium	0.276	(0.046)*	0.706	(0.038)*	0.39
Copper	0.241	(0.032)*	0.812	(0.019)*	8.74*
Zinc	0.210	(0.031)*	0.815	(0.020)*	2.03
Nickel	0.427	(0.049)*	0.700	(0.027)*	19.6*
Wheat	0.150	(0.022)*	0.865	(0.014)*	1.93
Corn	0.090	(0.013)*	0.901	(0.012)*	2.15
Soy	0.241	(0.030)*	0.718	(0.032)*	3.83*
Beef	0.315	(0.043)*	0.699	(0.031)*	0.82
Logarithm of price levels					
Oil	0.441	(0.050)*	0.617	(0.038)*	3.26
Coal	0.214	(0.039)*	0.755	(0.036)*	3.95*
Gas[d]	0.520	(0.063)*	0.491	(0.043)*	3.66
Aluminium	0.179	(0.041)*	0.748	(0.052)*	4.28*
Copper	0.065	(0.023)*	0.878	(0.035)*	10.51*
Zinc	0.064	(0.021)*	0.896	(0.028)*	6.35*
Nickel	0.196	(0.029)*	0.799	(0.031)*	0.76
Wheat	0.062	(0.015)*	0.933	(0.013)*	0.58
Corn	0.016	(0.010)	0.942	(0.039)*	1.75
Soy	0.113	(0.031)*	0.676	(0.086)*	9.35*
Beef	0.155	(0.049)*	0.613	(0.039)*	9.39*

*Statistically significant at 5% confidence level.
[a] Only estimates of parameters ρ and ω are reported. Other model parameter estimates are available on request.
[b] 1991M1-2021M12.
[c] 1992M1-2021M12.
[d] 1985M1-2021M12.

As expected, volatility clusters do not emerge for FoodInd, while a significant increase of volatility can be appreciated in the second part of the period of (starting around 2005) for both MetInd and EneInd. For these indexes, this volatility dynamics seems consistent with the increased price turbulence observed in the same period as shown in Figure A1. In the case of individu-

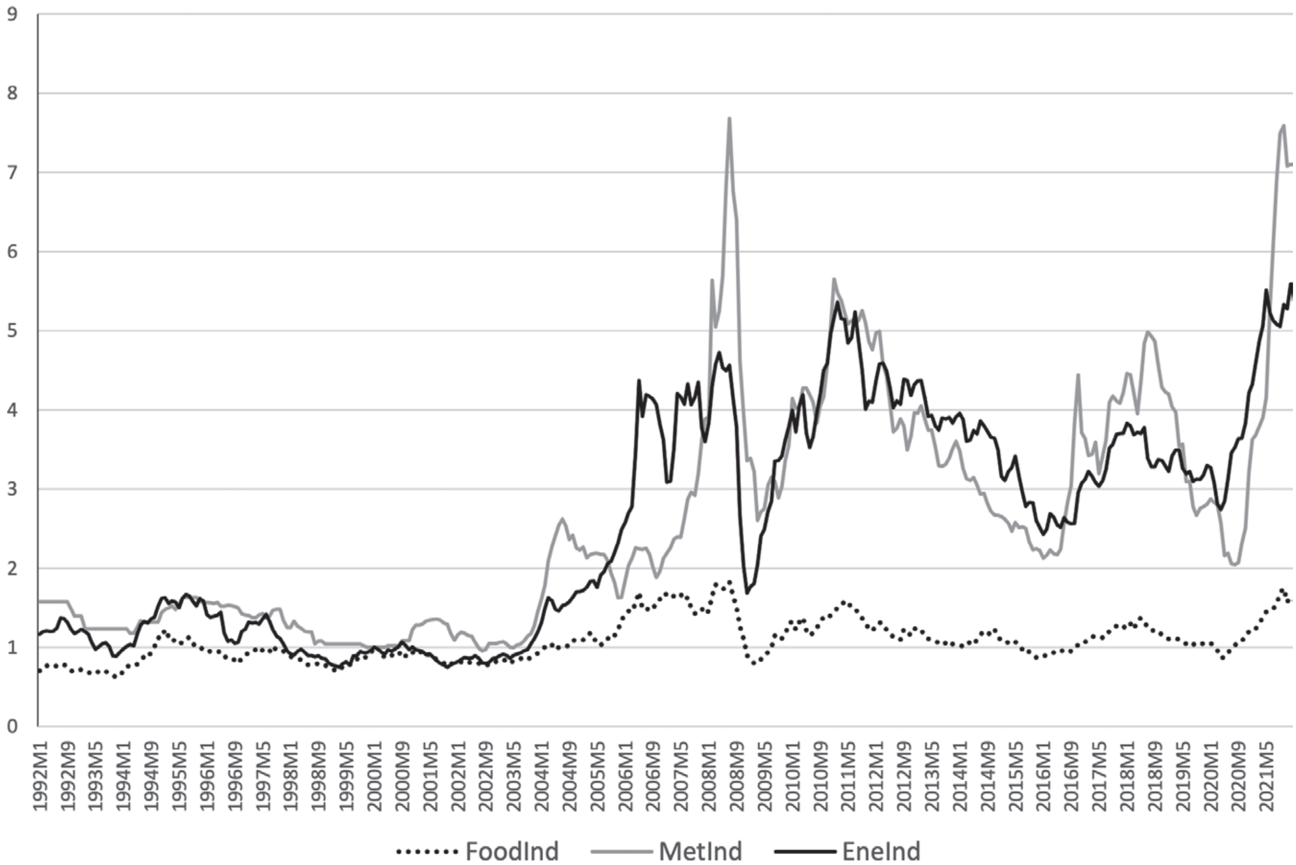


Figure 2. GARCH(1,1) model standard error in-sample prediction for commodity price indexes (2000M1=1) (1992M1-2021M12).

al series, predictions show huge volatility variations for oil and for all mineral and agricultural commodities. Clusters seem to be relatively rare and quite temporary in the first part of the period, while they become more frequent and longer, thus possibly permanent, from 2005 onwards. This seems even more true for CPI and therefore, but less intensively, for the inflation rate. CPI volatility sharply rises in 2005 and remains higher than in the previous period with only a drastic drop during years 2013-2014.

The question is whether the magnitude of this volatility clustering is consistent with the actual price turbulence or whether, in fact, we should look for alternative explanations.

5.3. *Temporary bubbles*

Table 3 reports the sequence of tests for the presence of temporary bubbles as expressed by equations (5) and (6). As discussed, moving from ADF_0^1 to GSADF the tests improve in terms of recursiveness and flexibility, therefore in precision, in detecting the temporary explosive roots.[16] The presence of a temporary bubble is excluded in all cases (price indexes, individual price levels and logarithms of individual price levels) when the search of the bubble extends to the whole period (ADF_0^1). Something emerges with SADF with a temporary explosive root observed for MetInd and EneInd, and for the price level of all energy commodities, all minerals, and wheat. In the case of the logarithm of prices a bubble is detected only for oil. The generalised occurrence of temporary bubbles is eventually indicated by the GSADF test. With the only exclusion of beef (both the price level and its logarithm), at least one temporary explosive root is found in all the series.[17]

[16] It is worth noticing that the ADF_0^1 test in Table 3 (second column) corresponds to the ADF test with drift in Table 1 (third column) as the explosive bubble tests associated to equation (4) may include a drift but not a deterministic trend. However, strictu sensu, they are not the same test since the former is a right-tailed statistics so the critical values are different. The statistics itself slightly differs in some cases because the adopted specifications (i.e., lag structure) are not always the same.

[17] Notice that the difference between the SADF and GSADF tests are larger here than what was presented in previous studies (see Gharib et al., 2021, p. 5, in particular) arguably because, despite the number

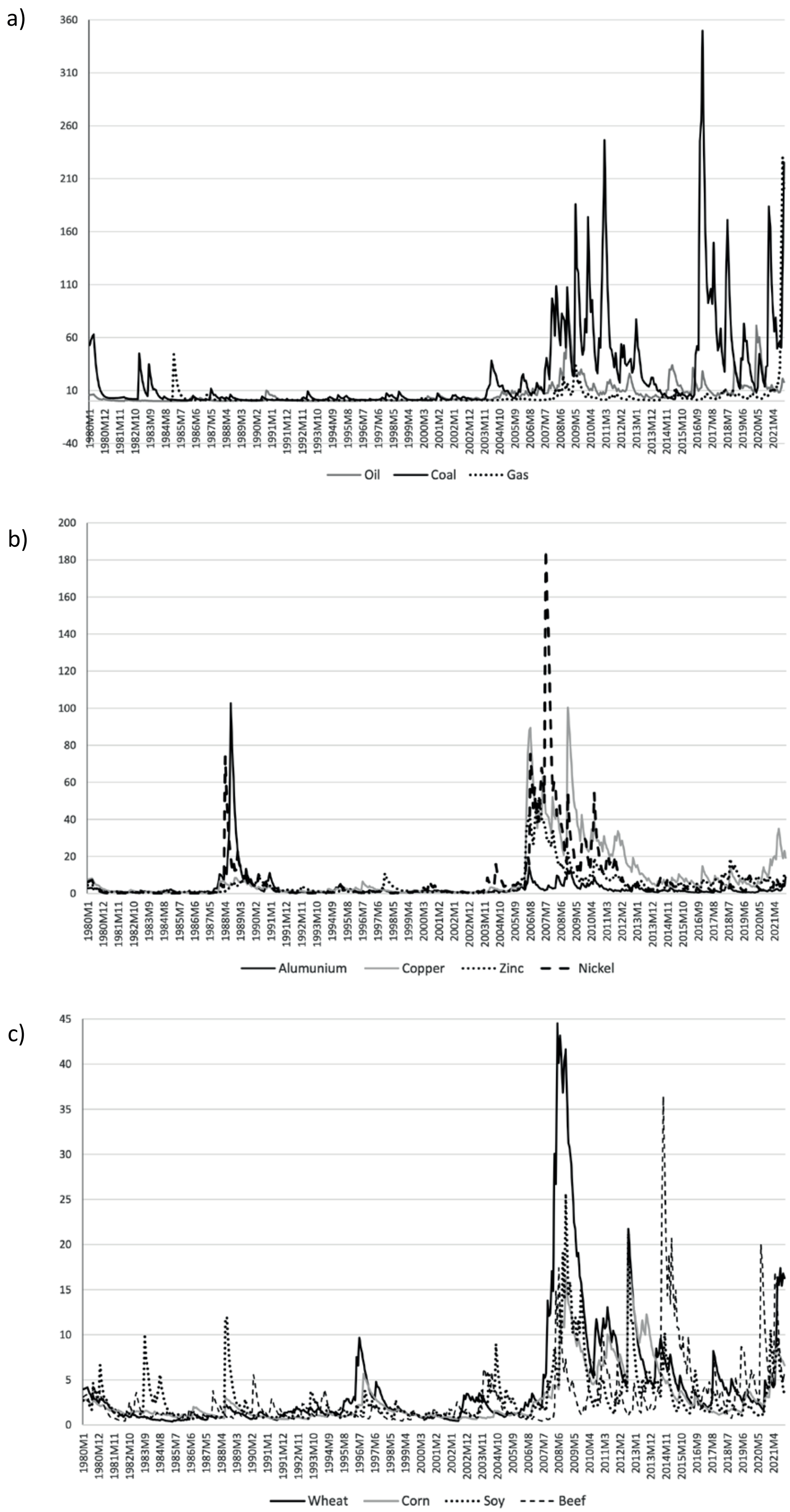


Figure 3a. GARCH(1,1) model standard error in-sample prediction for energy commodities (a), metals (b) and agricultural commodities (c) price levels (2000M1=1) (1980M1-2021M12).

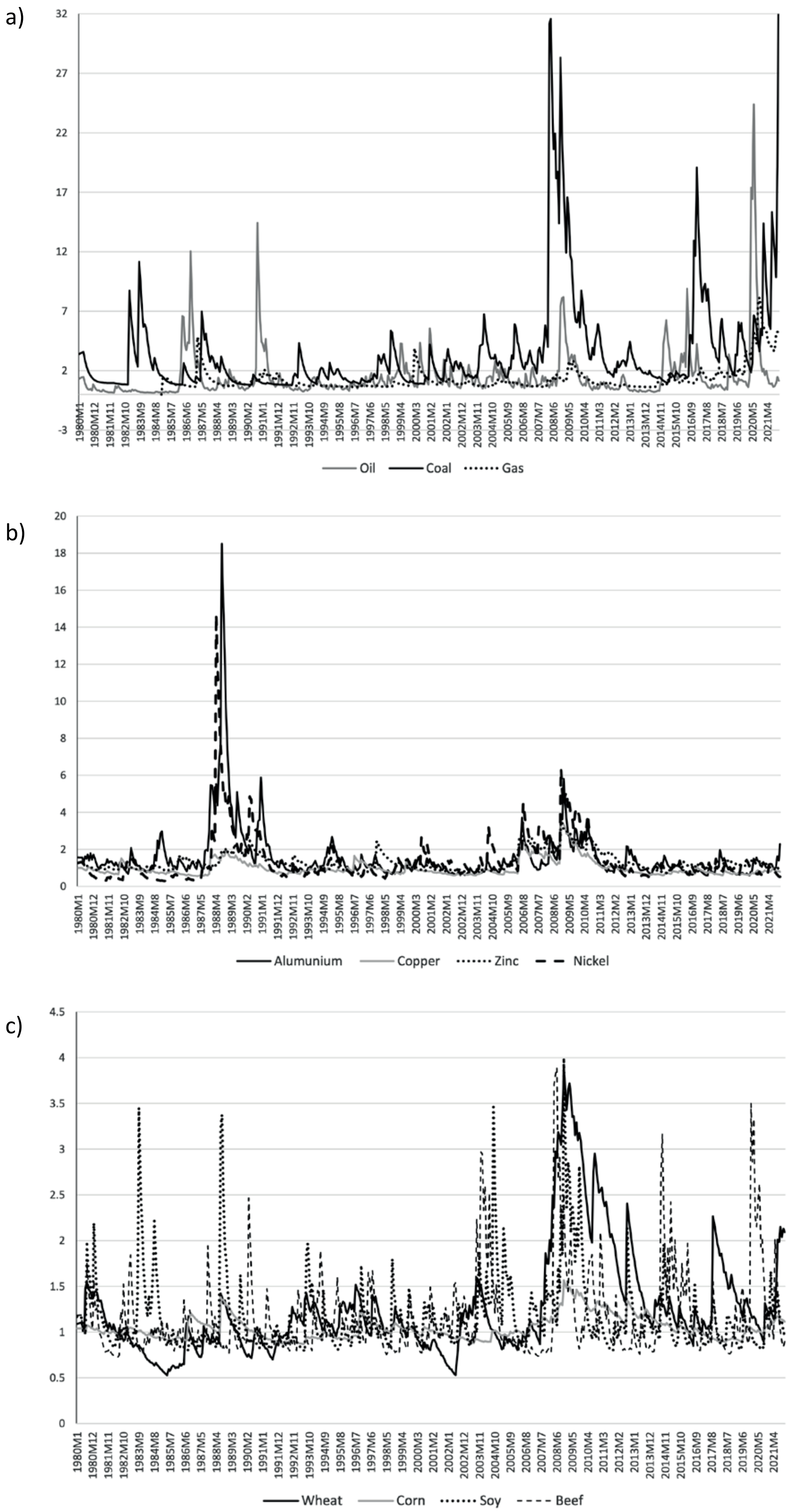


Figure 3b. GARCH(1,1) model standard error in-sample prediction for energy commodities (a), metals (b) and agricultural commodities (c) logarithm of prices (2000M1=1) (1980M1-2021M12).

Table 3. Temporary explosive root tests on commodity price indexes and commodity prices (1980M1-2021M12)[a].

Series	ADF_0^1	SADF	GSADF
Price indexes			
FoodInd[b]	-1.289	0.0487	3.644*
MetInd	-1.285	3.303*	7.170*
EneInd[c]	-1.167	4.907*	5.092*
CPI	0.288	0.854	3.189*
Price levels			
Oil	-2.352	3.843*	4.417*
Coal	-1.166	8.177*	8.762*
Gas[d]	-0.456	4.164*	6.619*
Aluminium	-2.524	2.405*	5.132*
Copper	-0.883	2.972*	4.750*
Zinc	-2.937	3.871*	5.908*
Nickel	-2.656	3.426*	5.491*
Wheat	-2.517	3.360*	3.795*
Corn	-2.151	0.357	3.462*
Soy	-2.478	-0.553	2.981*
Beef	-0.134	0.937	1.211
Logarithm of price levels			
Oil	-1.199	1.871*	2.275*
Coal	-1.500	0.414	2.511*
Gas[d]	-1.745	-0.135	2.891*
Aluminium	-3.168	-0.270	-2.816*
Copper	-0.858	-0.128	3.124*
Zinc	-1.827	0.057	3.583*
Nickel	-2.008	1.361	2.989*
Wheat	-2.410	1.120	3.098*
Corn	-1.756	0.513	2.306*
Soy	-2.032	-1.200	2.271*
Beef	-0.167	0.665	1.794

*Statistically significant at 5% confidence level with bootstrap critical values computed with 200 repetitions.
[a] All test specifications include 6 lags.
[b] 1991M1-2021M12.
[c] 1992M1-2021M12.
[d] 1985M1-2021M12.

In order to better appreciate how many bubbles occur and when, the BSADF tests (equation (7)) are computed. Results (with the critical values) are reported in Figure 4 for the three price indexes and in Figures 5a,b for the commodity price levels and logarithms, respectively. It appears that, for indexes, bubbles very sporadically emerge before and after the 2005-2008 period. On the contrary, over these four years the tests exceed the critical values several times for all the indexes. MetInd is the index for which this exceedance is more often observed.

In the case of individual price levels significant differences are found across the three groups. For energy prices, only in period 2005-2008 we observe one or more bubbles shared by the three prices. In the case of metals, beside that period, a common bubble is also observed in the mid-eighties. Agricultural commodities present a more composite situation: bubbles are more frequent and occur in the mid-eighties, mid-nineties, 2007-2008 and in the last decade. But they are often individual bubbles and, again, only in 2007-2008 we observe a bubble shared by most (except for beef) agricultural commodities. Qualitatively, results obtained with the price logarithms are similar even though, as could be expected, the bubbles are less frequent and, consequently, also the commonality of bubbles is more sporadic.

5.4. First movers and contagion

As discussed in previous sections, the focus of the present study is not on commodity price interdependence but on investigating the formation of temporary bubbles within individual price series in order to allow a real-time monitoring tool to inform on the formation of temporary bubbles, on the possible involvement of several commodities and on the first moving prices. The combination of the two alternative approaches here proposed allows to present their results in a form that permits an intuitive visualization of all this information about if and how co-exceedance is occurring.

Figures 6a,b aim to provide this easily interpretable visualization by displaying the periods of exceedance (volatility clusters or bubbles) for price levels and logarithms, respectively. Bubbles are dated on the basis of the BSADF tests. In the case of volatility, following Engle (1982, p 1003), exceedance is found any time the predicted volatility exceeds the double (in the case of price indexes) or the triple (in the case of individual commodity prices) of the average predicted volatility (i.e., standard error) over the two subperiods 1980M1-2000M12 and 2001M1-2021M12. Together with Table 4, these figures are also intended to provide an example of how the proposed approach can contribute to a real-time surveillance tool through an easily interpretable and periodically updatable dashboard visualization.

To summarize this evidence and better interpret it in terms of co-exceedance, Table 4 reports the beginning (the "exuberance date" to use the term of Gharib et al., 2021, p. 6) and the end (the "collapsing date") months of

of observations, the period covered here is quite long (more than 40 years).

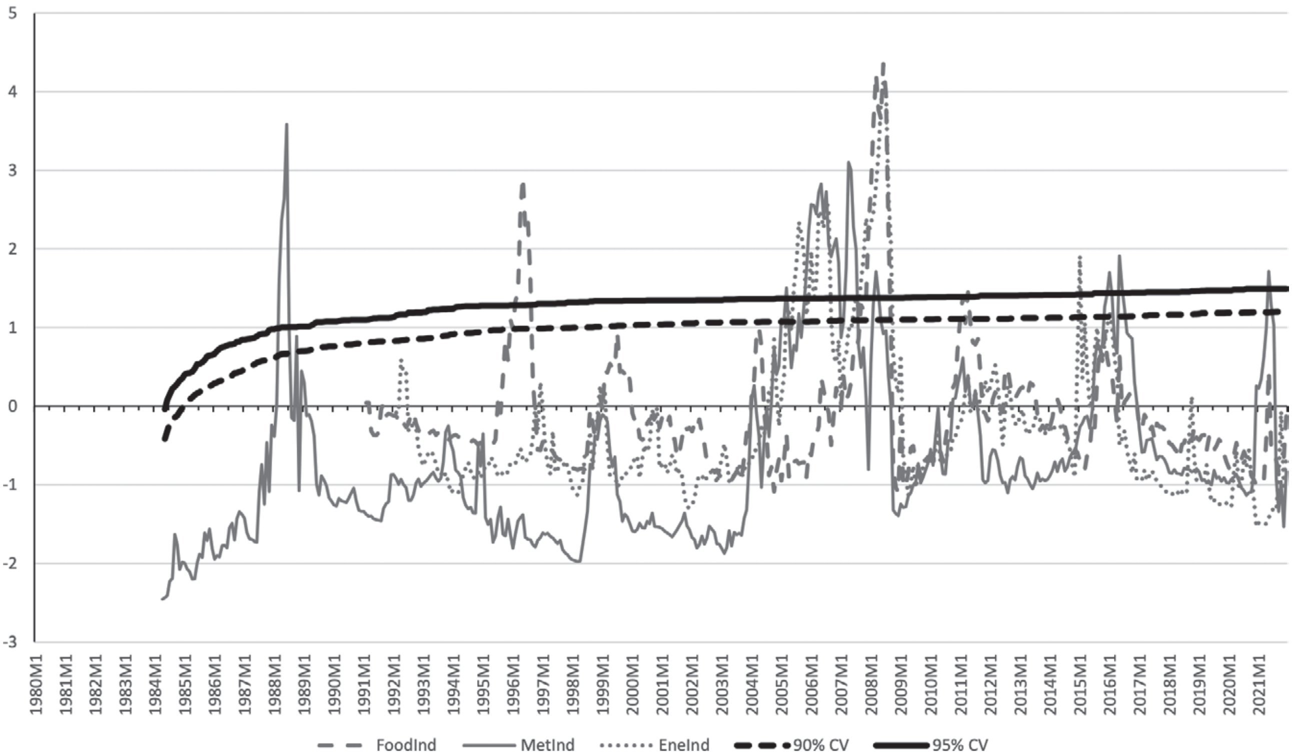


Figure 4. BSADF tests for indexes for commodity price indexes (1992M1-2021M12).

periods with at least three different commodities showing at least two consecutive months of exceedance, thus a co-exceedance in terms of volatility or common bubble for at least two consecutive months (Phillips et al., 2011). The table also reports: commodities showing this co-exceedance (second column); the commodity that can be identified as the first mover (third column), that is, the price whose exceedance started first before the period of co-exceedance; whether or not also CPI shows exceedance in the same period, and whether or not CPI can be considered the first mover (forth column).

In the case of price levels, it emerges that bubbles concentrate in the 2005-2008 period though in different moments involving different commodities. Basically, we can identify two main episodes. The first goes from 2005M8 to 2006M9. It only involves energy commodities and metals with oil and copper as first movers. CPI is itself involved in the bubble, as could be expected, but surprisingly it behaves as the first mover. The second episode is shorter and goes from 2008M2 to 2008M8. It involves all energy commodities wheat and corn but no minerals. Again, oil behaves as the first mover. This latter commodity actually seems to experience a single bubble from mid-2005 to the end of 2008. CPI is also involved but not as the first mover.

Volatility clusters emerging from the GARCH regressions show a significant difference compared to the bubbles. A first episode is found from mid-1988 to mid-1989, it concerns some minerals and agricultural commodities but no energy commodities. Wheat seems to be the first mover. Other four episodes, in fact behaving as a single one, can be detected from 2006M8 to 2009M8. In the first part of this period, the cluster exclusively involves metals and wheat. Then, other prices enter the group included energy commodities and, finally, also oil. In the very last part of this episode, the volatility clusters involve most (9 out of 11) commodities. If we consider this whole period as a single episode, the first mover seems to be nickel which sounds a little surprising. In the second part of the period, wheat and natural gas emerge as other possible candidates.

Two other volatility clusters can be found in the last decade of the period under investigation. One concerns a very short period (two months in mid-2012) and only involves agricultural commodities. The other concerns the very last months of the period of observation (2021M9-2021M12); it is short simply because it continues beyond the period of observation. This period of exceedance is not identified with the bubble testing arguably because the bubble has still to collapse. Future

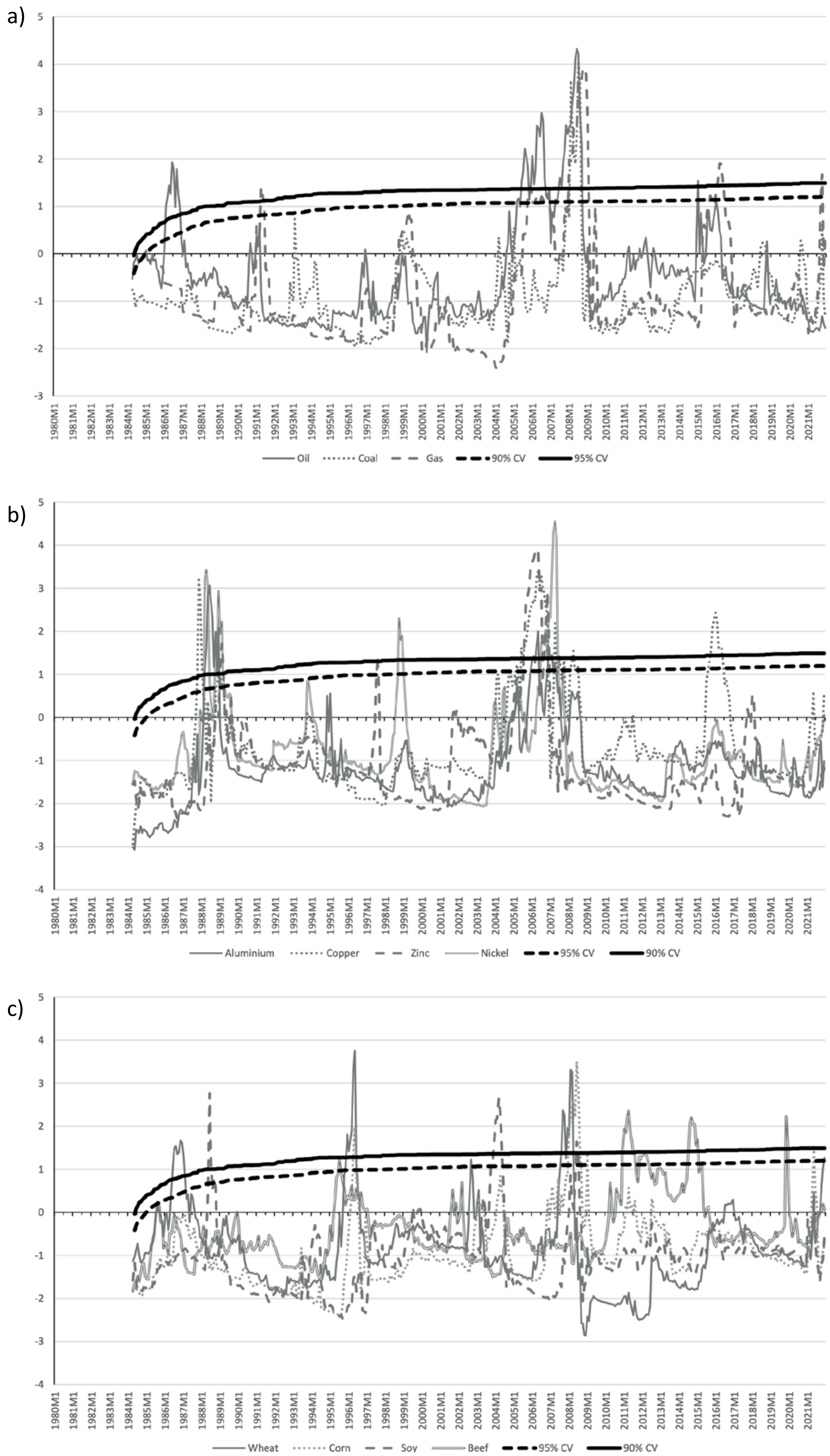


Figure 5a – BSADF tests for indexes for energy commodities (a), metals (b), agricultural commodities (c) price levels (1980M1-2021M12).

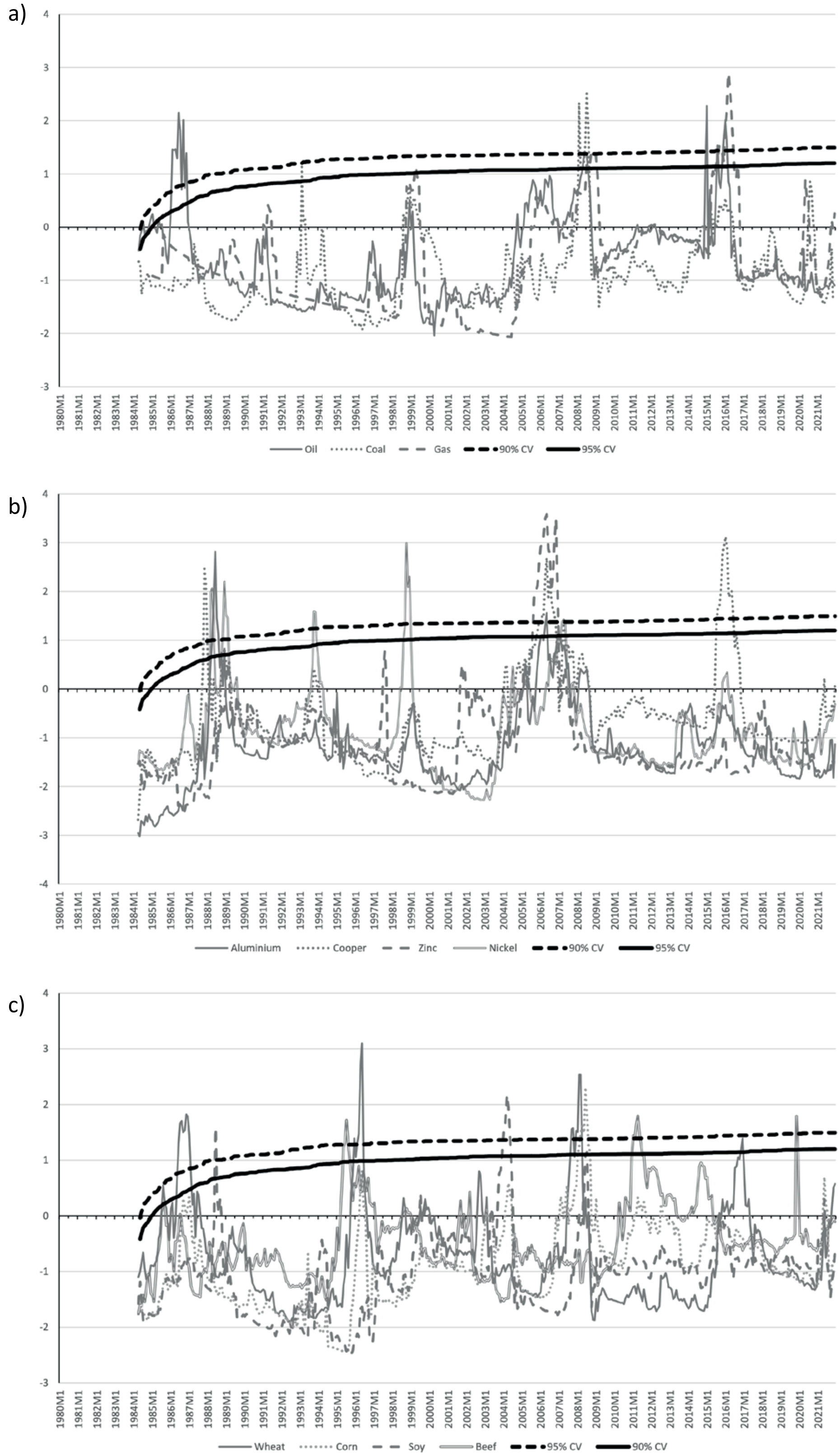
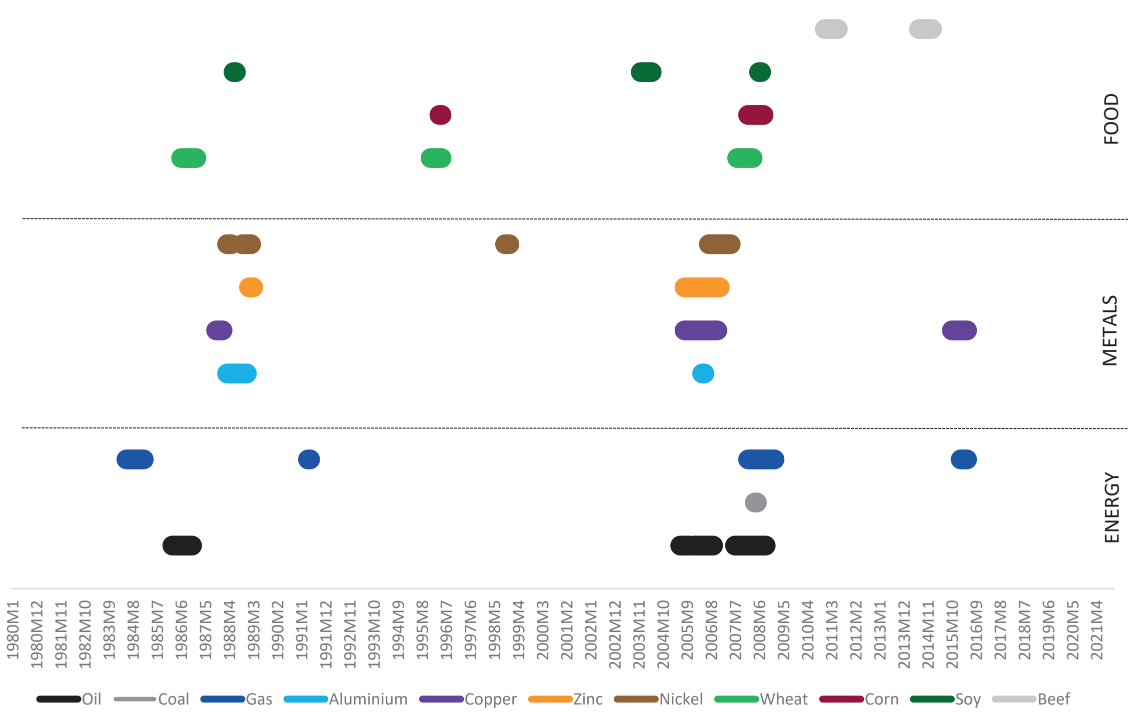


Figure 5b. BSADF tests for indexes for energy commodities (a), metals (b), agricultural commodities (c) logarithm of price levels (1980M1-2021M12).

a)



b)

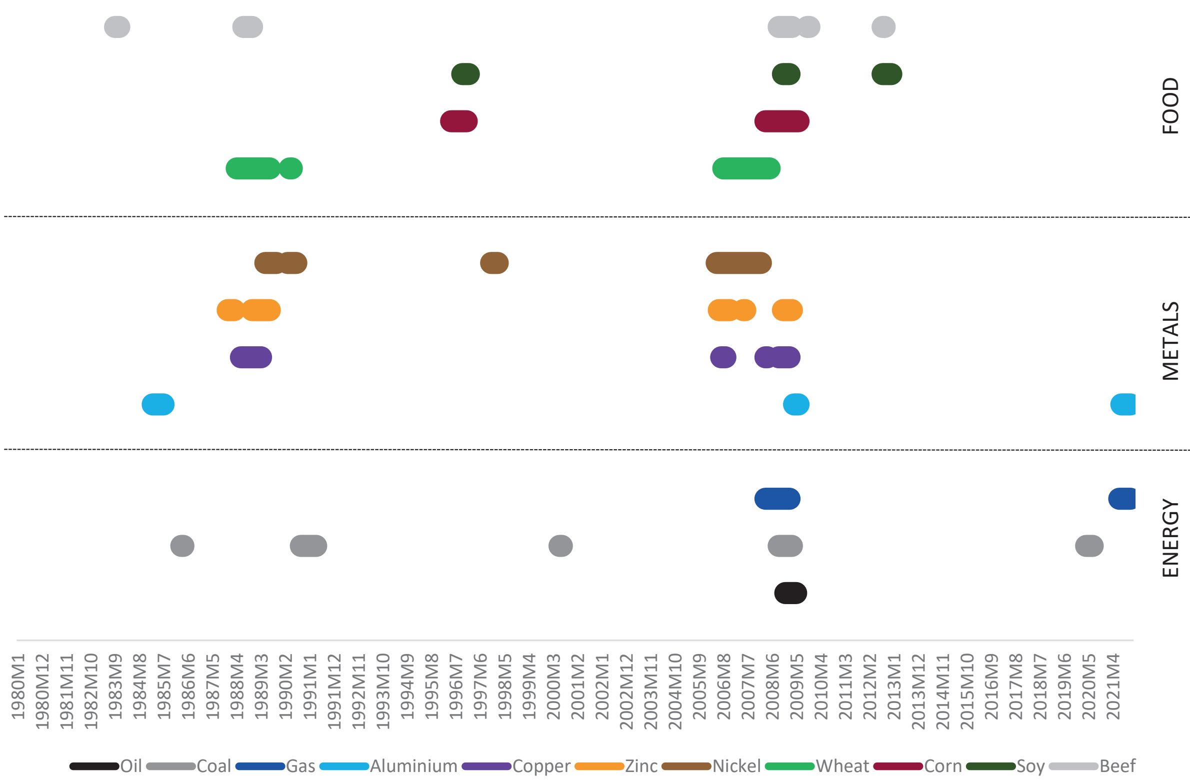


Figure 6a. Dating of explosive roots (a) and volatility clusters (b) for all commodity price levels (1980M1-2021M12) (see Table 4 for details).

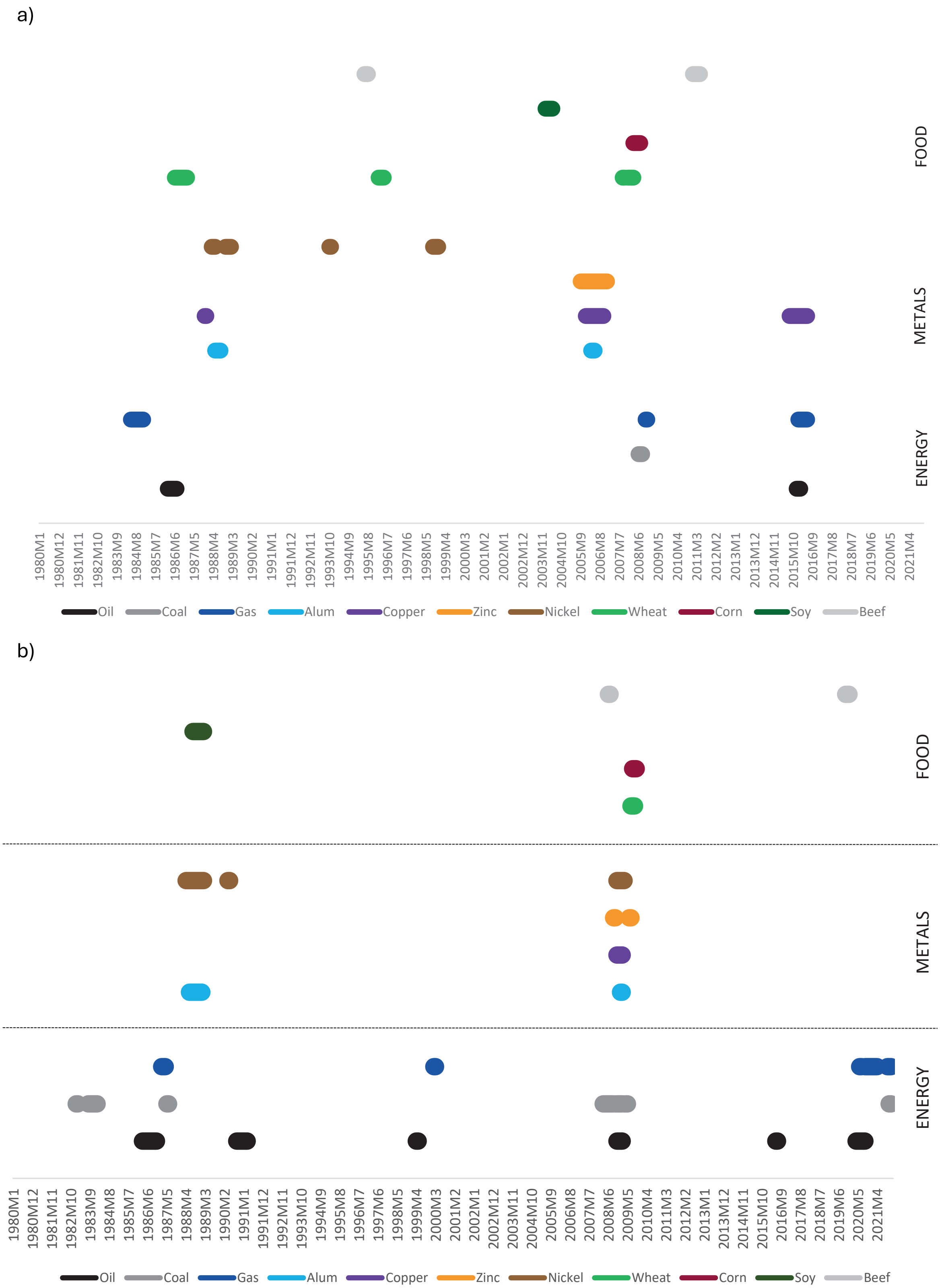


Figure 6b. Dating of explosive roots (a) and volatility clusters (b) for all commodity logarithms of price levels (1980M1-2021M12) (see Table 4 for details).

Table 4. Dating of temporary explosive roots and volatility clusters for commodity price levels and logarithm of levels (1980M1-2021M12).

Period	Commodities	First mover[c] (date)	CPI: Y/N; first mover (Y/N & date)[d]
	Price levels		
Bubbles[a]			
2005M8-2005M10, 2006M1, 2006M3, 2006M6	Oil, Copper, Zinc	Oil, Copper (2005M6)	Y; Y (2005M3)
2006M2, 2006M4-2006M5	Oil, Aluminium, Copper, Zinc		
2006M7, 2006M9	Oil, Copper, Zinc Nickel		
2006M10-2006M12	Oil, Copper, Zinc		
2008M2-2008M7	Oil, Coal, Gas, Wheat, Corn	Oil (2007M3)	Y; N
2008M5, 2008M8	Oil, Coal, Gas, Corn		
Volatility Clusters[b]			
1988M7-1988M10	Copper, Wheat, Beef	Wheat (1988M4)	N
1988M11-1989M7	Copper, Zinc, Nickel, Wheat		
2006M8-2006M11	Copper, Zinc, Nickel, Wheat	Nickel (2006M5)	N
2007M4-2007M12	Copper, Zinc, Nickel, Wheat		
2008M3-2008M8	Gas, Copper, Wheat, Corn	Wheat (2006M8)	N
2008M9-2009M8	Oil, Gas, Coal, Aluminium, Copper, Zinc, Corn, Soy, Beef	Gas (2008M3)	Y; N
2012M7-2012M8	Corn, Soy, Beef	Soy (2012M8)	N
2021M9-2021M12	Gas, Aluminium, Copper	Gas (2021M7)	Y; N/Y(2021M7)
	Logarithm of price levels		
Bubbles[a]			
Period	Commodities	First mover (date)	CPI: Y/N; first mover (Y/N & date)
2005M12-2006M10	Aluminium, Copper, Zinc	Zinc (2005M9)	Y; Y (2005M3)
2008M6-2008M7	Oil, Coal, Wheat, Corn	Wheat (2008M2)	Y; N
2015M12-2016M6	Oil, Gas, Copper	Copper (2015M8)	N
Volatility Clusters[b]			
Period	Commodities	First mover (date)	CPI: Y/N; first mover (Y/N & date)
1988M4-1989M1	Aluminium, Nickel, Soy	Aluminium (1987M12)	Y; N
2008M11-2009M3	Oil, Coal, Aluminium, Copper, Nickel	Coal (2008M3)	Y; N

[a] The dating of the bubble corresponds to periods when at least 3 commodities show explosive roots, that is BSADF test significant at 5% confidence level with bootstrap critical values computed with 200 repetitions. Only periods with at least two consecutive months of exceedance are reported.
[b] The dating of the volatility clusters corresponds to periods when predicted volatility (i.e., standard error) is larger than three times the subperiod (1980M1-2000M12; 2001M1-2021M12) average volatility. Only periods with at least two consecutive months of exceedance are reported.
[c] The first mover is the price of the group whose exceedance started first before the period of co-exceedance.
[d] The first Y/N indicates whether or not also CPI shows exceedance in the same period; the second Y/N indicates whether or not CPI can be considered the forst mover (in parenthesis the date).

investigations will confirm the nature and scope of the current period of exuberance. The cluster identified here suggests that it concerns both energy prices and metals but it is likely driven by the former and, in particular, by natural gas.

Two major facts seem to emerge from this analysis of co-exceedance. First of all, the correspondence between bubbles and volatility cluster detection is limited. Periods correspond in the case of the major episode that occurred between 2005 and 2008. But for the rest of the sample, the detection of the episodes of co-exceedance does not correspond. Also the involved commodities significantly differ and, consequently, the first movers. Bubble detection seems to stress more the dynamics of energy commodities, and oil in particular, while volatility clusters point more to metals and agricultural commodities. A final difference concerns the involvement of CPI that seems very limited in the case of clusters while it is more relevant for bubbles.

The second notable fact is the difference between price levels and price logarithms. As the logarithmic transformation re-scales the data and, therefore, scales down their variability, respective results are expected to make the more robust evidence emerge: the number of individual episodes may slightly decline and the number of common episodes is expected to substantially reduce. It turns out that the number of episodes of co-exceedance detected on price logarithms is lower, as expected, for both bubbles and volatility clusters. But the nature of these episodes does not necessarily correspond with what is observed on price levels and this lack of robustness passing from levels to logarithms seems more evident for bubbles than for volatility clusters. The involved commodities are not necessarily the same, as well as the first movers, and also the involvement of the CPI shows some difference. In general terms, when the logarithms are considered, it seems more difficult to find some general pattern in the results, especially in terms of a key role of some commodities like the energy ones.

Looking for regularities, two special cases are worth noticing here. The first concerns the oil price. Properties and behaviour of this commodity price emerging in the present work confirm the bubble detection and dating reported in previous studies, particularly in Su et al. (2017) and Zhao et al. (2021).[18] It could also be argued that oil price has to behave as a sort of upstream price since it enters as a production cost in most downstream production processes, included farming and mining activities. But this role of oil as first mover is not generally observed and seems to emerge only in bubble testing with price levels.

The second interesting role is that played by agricultural commodities. On the one hand, they can be considered as downstream prices compared to energy commodities and metals. But, for this reason, they can severely impact on CPI dynamics. From our results, it emerges that agricultural commodity prices seem to be a little more "stable" in the sense that episodes of exuberance (bubbles or volatility clusters) are less frequent and shorter. At the same time, while energy commodities and metals are apparently more interdependent, agricultural prices seem to follow more autonomous patterns and are less likely to act as first movers and, thus, to be suitable candidates to drive the other commodity prices and of the CPI.

What can we finally conclude about the evidence on the linkage between commodity prices and CPI? While results tend to confirm some stochastic properties of the CPI that may explain periods of exuberance, the evidence that these periods are the consequence of analogous episodes in commodity prices is poor. The major episode of price exuberance between 2005-2008 confirms, as could seem obvious, a connection between commodity prices and CPI, maybe because this episode involves a large number of commodities, though in different times. In fact, this connection seems quite weak beyond this period. And also within this 2005-2008 period it is not clear whether commodity price exuberance induced a CPI response or if it is actually the other way round. This lack of evidence should not be surprising and evidently asks for further investigation. Other very recent empirical investigations (Lian and Freitag, 2022), for instance, suggest that oil price shocks do not always imply a shock on CPI and sometime this latter may move independently and also precede the former.

6. COMPARISON WITH OTHER STOCHASTIC PROCESSES AND APPROACHES

For the sake of comparison and in order to validate the results here obtained, it is worth investigating the commonality of the commodity price dynamics also with more conventional approaches. Rather than focusing on co-exceedance, as in the present study, these approaches look for the commonality of the stochastic generation processes (i.e., co-movement and the consequent price interdependence) under the typical hypothesis of either stationary or non-stationary linear DGP, possibly with a drift and/or a trend (Esposti, 2021). As already discussed in Section 2, in order to capture the complexity and non-linearity of these series a further occurrence that can be considered consists in admitting that series undergo, in one or more points in time, a structural break in either the drift or the trend (or in both) (Baum, 2005). In principle, under multiple breaks, these stochastic processes could explain the presence of periods of extremely high (or low) prices (the "bubbles") as a sequence of two structural breaks with the latter eventually compensating the former and thus making its effect only temporary.

Table A2 (Annex 3) reports a battery of tests specifically designed to assess whether these more conventional stochastic processes represent suitable alternatives to the two co-exceedance processes here considered. Four tests are reported. They all confront a unit-root process (the null hypotheses of the tests) with a stationary process

[18] Su et al. (2017, p. 6) conclude that "there are explosive multiple bubbles in the WTI oil market in 1990, 2005, 2006, 2008 and 2015. Generally, oil bubbles mostly occur during the period of price volatility".

presenting one or two structural breaks in some terms of the process itself.[19] All tests admit endogenous breaks, thus not only do they test their presence but they are also able to date these breaks.

The first two tests consist in two specifications of the Zivot-Andrews (ZA) unit root test (Zivot and Andrews, 2002) admitting only one structural break in either the intercept or both intercept and trend. Test results largely accept unit-root processes without a break against the presence of a structural break within stationary series. The only exceptions are coal and soy in the price levels and only soy in the logarithms of prices. The break date is similar, late 2006-early 2007, and corresponds to one of the major periods of co-exceedance identified in previous sections. Not only is the break accepted for only two commodities but, more importantly, it can not explain why and how, once started, the period of turbulence then comes to an end since the structural break introduces a permanent change in the process.

The other two tests, consisting in two variants of the Clemente-Montañés-Reyes unit root test (CMR) (Clemente et al., 1998), can be helpful in this respect. In this case, the statistical significance of the breaks themselves can be assessed as they enter in the test specification as time dummies with the respective coefficients. More importantly, this test admits two structural breaks within the stationary process thus allowing the combination of the two breaks to capture a temporary change in the process, like in the case of periods of price surge. The test can be performed under two different natures of the breaks: a sudden change in the series (the additive outliers, or AO, model) or a gradual shift in the mean of the series (the innovational outliers, or IO, model). Evidently, the former, much more than the latter, is expected to capture the short periods of price turbulence.

Although CMR tests confirm how difficult it can be, over such a long period of time, to univocally identify clear stable stochastic processes for any given commodity and, even more, commonality across commodities in this respect, they still provide some interesting indications. One the one hand, the CMR test under the AO model seems to confirm the main evidence emerging from the ZA test results without any relevant difference between price levels and logarithms: for most commodity prices (oil being the only exception) a non-stationary process is accepted against a stationary process with structural breaks. When the IO model is considered, however, the CMR test indicates that for many commodities (all energy prices, aluminium, zinc, corn and FoodInd itself) a mean-reverting process under two structural breaks is accepted. Even more interestingly, this test indicates that, for both AO and IO cases, the structural breaks are always statistical significant (with only one exception). In some cases, the interval between the two breaks is too wide (more than three years) to really capture a period of price exuberance (see the CPI case, for instance). In other cases, however, the time window between the two breaks seems quite consistent with the periods of exceedance here identified, as shown in Figures 6a,b. This is the case, in particular, of coal and all metals.

As already discussed, with respect to the purpose of the present study, the introduction of structural breaks may seem an unnatural way to capture co-exceedance: it still maintains the linear specification of the DGP possibly with a permanent change while here the intention is to identify a DGP with temporary non-linearities. It follows that admitting structural breaks within the stochastic process representation may still confound short-term and long-term dynamics within the price series. Nonetheless, present results suggest that multiple breaks within an appropriate specification eventually constitute a sort of spline process capable to proxy temporary non-linearities. Even though not considered further here, this kind of approach, together with the introduction of multiple structural breaks within non-linear processes (Bai and Perron, 2003; Caporale et al., 2020), can represent a promising alternative empirical strategy in future research in the field.[20]

There is a final aspect to be considered about the introduction of structural breaks as a valid alternative to capture co-exceedance. It concerns the identification of the first-moving commodities and the possible consequent contagion process. As shown, within the proposed approach, this identification is made only qualitatively by identifying and then visualizing when, commodity by commodity, the periods of exuberance start and end (see Figures 6a,b). Very often, however, within the empirical literature this identification is formally pursued using Granger causality testing (Esposti and Listorti, 2013). This approach must satisfy the prerequisite that series under investigation show the same stochastic properties (they are all either I(0) and I(1)), and then it requires the estimation of multiple-equation linear models (in the form of VAR or VEC models, respectively) representing the common movement from which direction and nature of price interdependence (or transmission) can be assessed. Within this representation one or more struc-

[19] For more details on the ZA and CMR tests, also see Baum (2005).

[20] The use of international or global commodity prices, as well as the widely heterogenous dating of these structural breaks across commodities, makes it hard to speculate on the possible linkage between them and external shocks like, for example, policy regime changes. However, this investigation may represent a further direction of research for the future.

tural breaks can be included (as time dummies) to possibly capture some changes in the linear relationships, thus admitting temporary non-linearities.

Results here presented demonstrate how the prerequisite of this empirical strategy to detect first movers and contagion can be challenging. A common DGP is impossible to find when all price series are considered. But even concentrating on individual commodity groups, Tables 1 and 5(??) suggest there is always at least one commodity showing a different underlying stochastic process compared to the others. Apparently, an interesting case is that of energy commodities under the IO specification of the CMR test: when two structural breaks are admitted they all behave as stationary processes with a drift and a deterministic trend. Therefore, an attempt to perform Ganger causality testing can be made here by estimating a VAR model with four endogenous variables: the three energy prices (oil, coal and gas prices) and the CPI, since it quite robustly emerges as a I(0) process. The VAR specification also includes a drift, a deterministic trend and two time dummies representing the two breaks at 2003M11 and 2013M5. Table A3 (Annex 3) reports the results of the respective Granger causality tests and the estimated coefficients of the two structural breaks.[21]

As often occurs with Granger causality testing, results are not easily interpretable. However, they confirm some of the evidence obtained with our proposed approach. It seems hard to identify an indisputable driving price and, in particular, this does not seem the case of the oil price. When price levels are considered, oil and coal show a reciprocal Granger causation, while natural gas is only Granger caused by the coal price. Oil and coal price both Granger-cause the CPI response, while CPI itself does not Granger-cause any of the energy prices as could be expected. Coal rather than oil seems to be the driving price, if any, and this seems to be reinforced when the logarithm of prices are considered instead of the levels. The presence of structural breaks, though suggested by tests reported in Table A2, is not confirmed by VAR estimation coefficients associated to respective time dummies are mostly not statistically different from zero.

Compared to the approach here proposed, which is based on the search of co-exceedance periods (thus admitting non-linearities in the DGP) rather than on linear price interdependence, these more conventional stochastic processes do not seems to provide any helpful additional information. On the contrary, they seem to fail in the search of common periods of exuberance over a large group of commodities, thus they do not seem appropriate for designing a real-time surveillance dashboard informing a prompt policy response. Nonetheless, even in these approaches recent contributions have opened new interesting perspectives that may deserve careful consideration in future research. For instance, the implementation of non-linear Granger causality testing seems particularly promising (Shahzad et al., 2021).

7. CONCLUSIVE REMARKS

Periods of commodity price exuberance raise political concerns particularly for their possible impact on the inflation rate. Timely interventions by the appointed institutions are often invoked but do not necessarily prove to be effective in preventing or neutralizing these episodes. After all, common price spikes (thus, co-exceedance) might not imply a common policy response since for some commodities exuberance tends to be motivated by real drivers while in other cases financial phenomena are prevalent. Understanding the mechanisms underlying generation, transmission and, then, collapse of co-exceedance remains relevant to design the proper, possibly differentiated, policy response. But in the shorter term an appropriate policy response may just need a timely detection of the price surge and of the degree of diffusion across commodities.

The present paper aims to develop a single methodological approach, albeit based on alternative stochastic processes, that does not assume common movement and price interdependence but only co-exceedance, thus commonality occurring only within the periods of exuberance. This approach is able to detect whether such a period occurs, when it starts and when it ends, the degree of diffusion across commodities, the possible presence of driving prices and, eventually, the transfer to the inflation rate. On this basis, the proposed methodology is intended to offer an easily interpretable visualization of the critical information it generates.

Results presented indicate that the different approaches considered (bubbles and volatility cluster detection in both price levels and logarithms) are able to provide clear indications on when the exceedance occurs, on its overlapping across commodities and on possible first movers. However, this evidence is not concordant or, at least, robust across the different approaches making the final outcome of the analysis, and the policy implication itself, severely dependant on the analyst's choices in this respect. Results do not even agree on the involvement of the CPI in these episodes of exuberance, therefore on the transmission of commodity price spike to inflation rate.

[21] For the sake of space limitation, the VAR model estimates are not reported here but are available upon request.

On the basis of this discrepancy, it seems wise to develop the abovementioned policy tool in a way that prudently admits both processes and elaborates information from a combination of them. At the same time, this discrepancy points to room for further methodological improvements. After all, both competing representations of the origin of exceedance, volatility clusters and temporary bubbles, show pros and cons and this makes it difficult to draw a general preference for one or the other. GARCH modelling seems to represent more permanent changes in volatility rather than short periods of exuberance. Furthermore, it hardly combines volatility clusters with a non-stationary process in the price levels. At the same time, bubble detection applies well to positive bubbles, therefore periods of exuberance then followed by a collapse, but it does not necessarily succeed in case of negative bubbles, that is episodes that start with a price crash (Gharib et al., 2021, pp. 3-4).[22] In fact, bubble detection can only by applied *ex post*, therefore when the bubbles have already collapsed. This substantially limits the actual applicability of the approach by analysts and policy makers. Moreover, currently available tests only apply to univariate bubble detection. Multivariate bubble testing has not yet been proposed and this prevents a direct investigation of contagion across commodities.

Regarding all these aspects, results obtained in the present study also suggest the extension of the adopted tool to other stochastic processes, particularly those expressing non-linear dynamics of commodity prices in both level and variance. Multiple breaks, fractional integration, fractal and wavelet analysis are some examples in this direction. Finally, it would be particularly helpful to replicate these results on higher frequency price data. Weekly or daily prices, if available, might definitely be useful to better refine this real-time surveillance policy tool making it more timely and accurate. However, these data might also bring about more statistical noise, thus making the identification of co-exceedance more difficult and uncertain, and increasing the risk of false alarms. Therefore, replication of the present analysis on these data could allow to assess the advantages and disadvantages in the use of higher frequencies.

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ANNEX 1 - DESCRIPTION OF THE DATA USED IN THE ANALYSIS

Individual commodity prices (source: IMF)

Oil: Crude Oil (petroleum), Price index, 2005 = 100, simple average of three spot prices; Dated Brent, West Texas Intermediate, and the Dubai Fateh
Gas: Natural Gas, Russian Natural Gas border price in Germany, US$ per Million Metric British Thermal Unit
Coal: Australian thermal coal, 12,000- btu/pound, less than 1% sulfur, 14% ash, FOB Newcastle/Port Kembla, US$ per metric ton
Aluminium: 99.5% minimum purity, LME spot price, CIF UK ports, US$ per metric ton
Copper: grade A cathode, LME spot price, CIF European ports, US$ per metric ton
Zinc: high grade 98% pure, US$ per metric ton
Nickel: melting grade, LME spot price, CIF European ports, US$ per metric ton
Wheat: No.1 Hard Red Winter, ordinary protein, Kansas City, US$ per metric ton
Corn: U.S. No.2 Yellow, FOB Gulf of Mexico, U.S. price, US$ per metric ton
Soy: U.S. soybeans, Chicago Soybean futures contract (first contract forward) No. 2 yellow and par, US$ per metric ton
Beef: Australian and New Zealand 85% lean fores, CIF U.S. import price, US cents per pound

Table A1. Descriptive statistics on commodity price indexes and commodity prices (1980M1-2021M12).

Series	Obs	Minimum	Maximum	Mean	Standard Deviation	Skewness	Kurtosis
Price indexes							
FoodInd[a]	372	76.04	194.1	121.9	60.20	-0.384	1.965
MetInd	504	44.16	256.2	101.3	53.69	-0.084	2.556
EneInd[b]	360	22.07	257.3	99.75	67.76	-0.656	2.244
CPI	504	40.79	146.2	92.17	28.09	-0.005	1.799
Price levels							
Oil	504	18.44	249.6	84.09	56.86	0.877	2.534
Coal	504	24.09	240.7	57.66	34.01	1.584	5.816
Gas[c]	444	1.444	32.91	5.398	4.199	1.979	10.42
Aluminium	504	918.8	3578	1712	468.1	0.778	3.325
Copper	504	1272	10308	3882	2520	0.742	2.116
Zinc	504	597.4	4381	1541	786.4	1.019	3.277
Nickel	504	3433	51783	11394	7270	1.867	8.130
Wheat	504	88.55	403.8	168.5	54.33	1.235	4.396
Corn	504	65.35	332.9	142.7	57.14	1.420	4.542
Soy	504	158.31	622.9	292.1	105.4	1.036	3.204
Beef	504	74.26	272.2	130.5	44.44	1.026	3.111
Price logarithms							
Oil	504	2.914	5.520	4.207	0.671	0.202	1.755
Coal	504	3.178	5.483	3.913	0.510	0.610	2.358
Gas[c]	444	0.367	3.493	1.461	4.250	1.255	6.286
Aluminium	504	6.823	8.182	7.409	0.265	0 .158	2.542
Copper	504	7.148	9.240	8.056	0.642	0.289	1.566
Zinc	504	6.392	8.385	7.222	0.477	0.370	1.948
Nickel	504	8.141	10.85	9.173	0.566	0.330	2.406
Wheat	504	4.483	6.000	5.081	0.296	0.489	2.882
Corn	504	4.179	5.807	4.894	0.353	0.663	2.868
Soy	504	5.064	6.434	5.619	0 .332	0.520	2.277
Beef	504	4.307	5.606	4.820	0.313	0.568	2.316

[a] 1991M1-2021M12.
[b] 1992M1-2021M12.
[c] 1985M1-2021M12.

Aggregate commodity price indexes (source: IMF)

FoodInd: Food Price Index, 2016 = 100, includes Cereal, Vegetable Oils, Meat, Seafood, Sugar, and Other Food (Apple (non-citrus fruit), Bananas, Chana (legumes), Fishmeal, Groundnuts, Milk (dairy), Tomato (veg)) Price Indices
MetInd: Metals Price Index, 2005 = 100, includes Copper, Aluminium, Iron Ore, Tin, Nickel, Zinc, Lead, and Uranium Price Indices
EneInd: Fuel (Energy) Index, 2005 = 100, includes Crude oil (petroleum), Natural Gas, and Coal Price Indices

Overall Consumer Price Index (source: US Federal Reserve)

CPI: Federal Reserve Economic Data, Economic Research Division, Federal Reserve Bank of St. Louis. CPIAUCNS Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, Index 1982-1984=100, Monthly, Not Seasonally Adjusted. The **Inflation rate** is computed as the monthly growth rate of this CPI.

ANNEX 2 – COMMODITY PRICE DYNAMICS

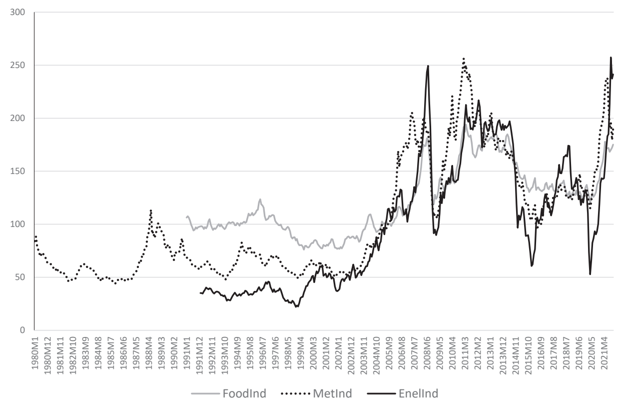

Figure A1. Commodity price indexes (2005M1=100) (1980M1-2021M12).

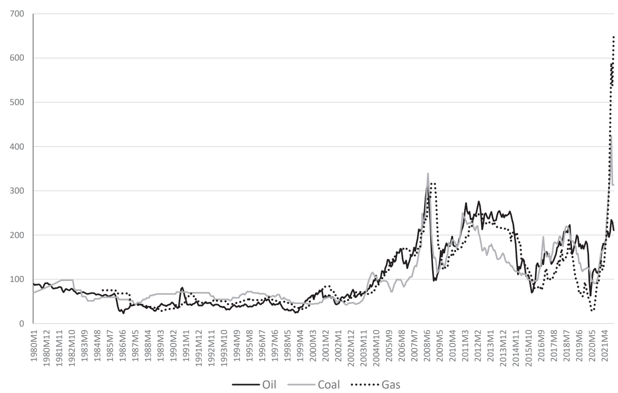

Figure A2. Energy commodities prices (2005M1=100) (1980M1-2021M12).

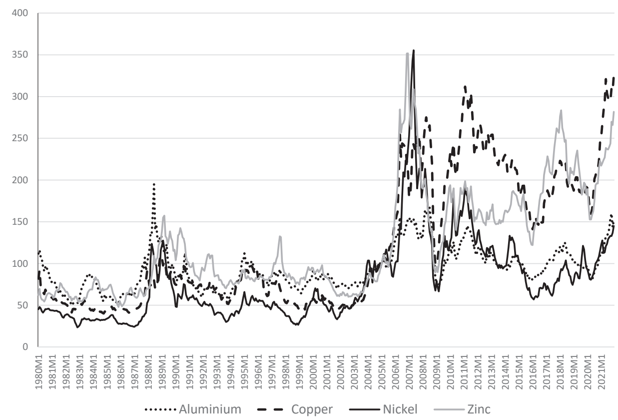

Figure A3. Metals prices (2005M1=100) (1980M1-2021M12).

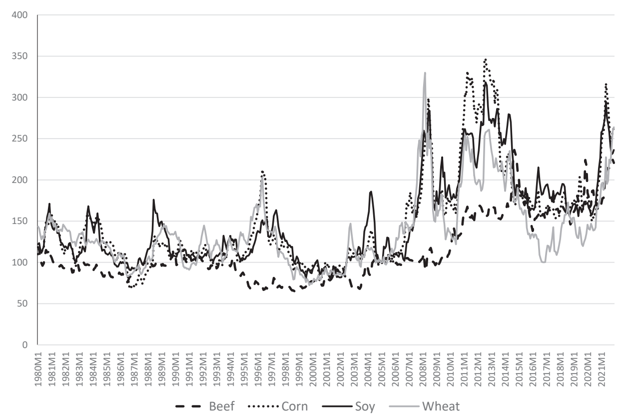

Figure A4. Agricultural commodities prices (2005M1=100) (1980M1-2021M12).

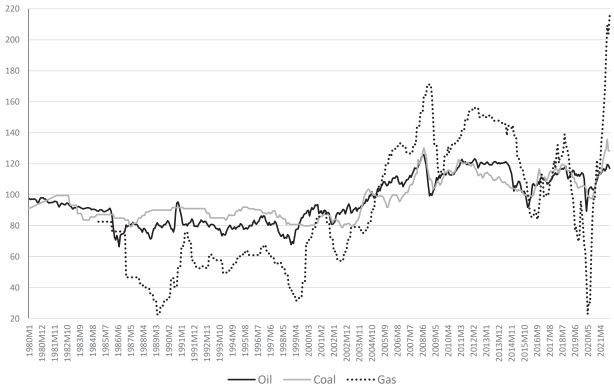

Figure A5. Logarithms of the energy commodities prices (2005M1=100) (1980M1-2021M12).

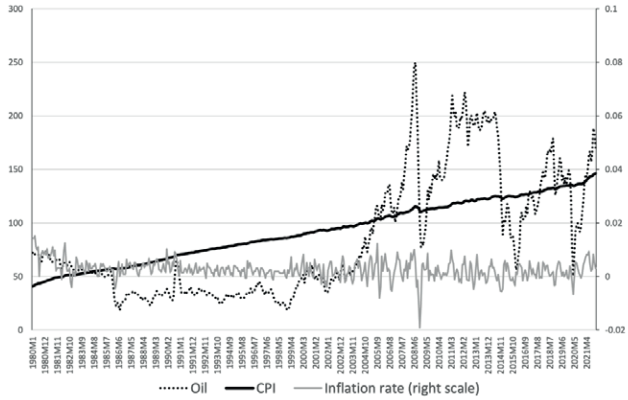

Figure A8. Oil price (2005=100), CPI (2005=100) and inflation rate (1980M1-2021M12).

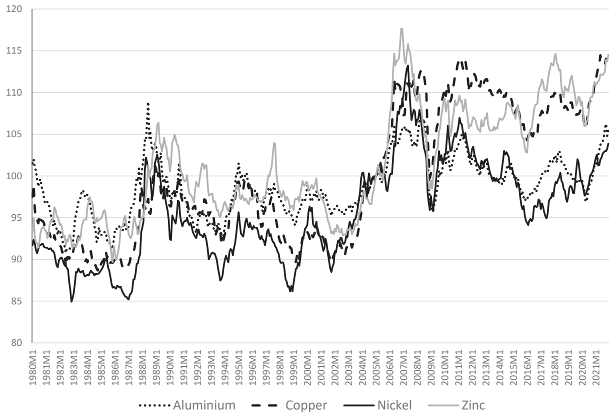

Figure A6. Logarithms of the metals prices (2005M1=100) (1980M1-2021M12).

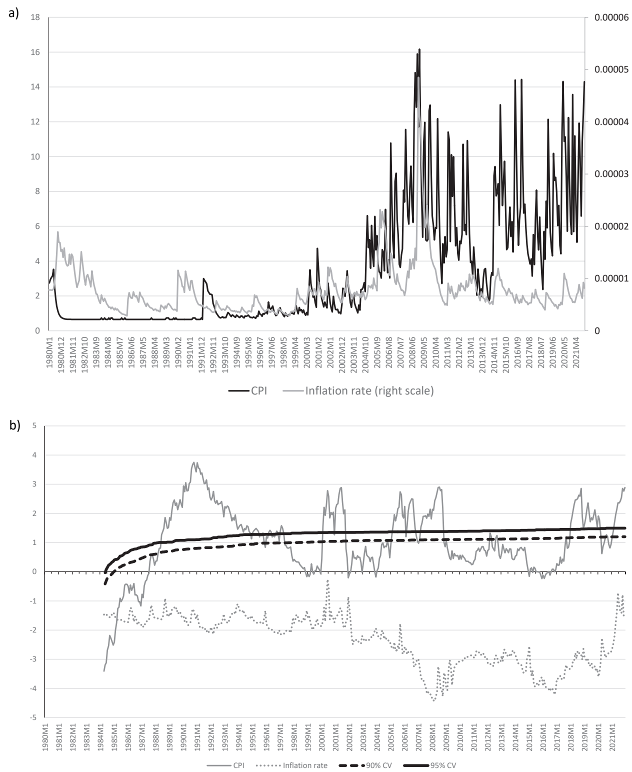

Figure A9. GARCH(1,1) model standard error in-sample prediction (a) and BSADF test for CPI and Inflation rate (1980M1-2021M12).

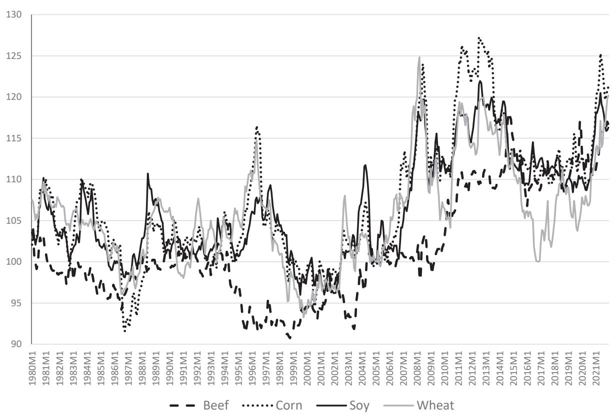

Figure A7. Logarithms of the agricultural commodities prices (2005M1=100) (1980M1-2021M12).

ANNEX 3 – UNIT-ROOT AND GRANGER-CAUSALITY TESTS WITH STRUCTURAL BREAKS

Table A2. Testing with structural breaks within an ADF specification (unit-root testing) for commodity price indexes, price levels and logarithm of levels (1980M1-2021M12).

Series	ZA (break month)[e]		CMR (b.reak months)[f]	
	Intercept	Intercept&Trend	AO	IO
Price indexes				
FoodInd[a]	-4.780	-4.932	-3.844 (1990M2*; 2006M3*)	-7.408* (1990M2*; 2006M3*)
MetInd	-3.826	-4.048	-5.176 (2005M6*; 2014M3*)	-5.406 (2004M9*; 2013M7*)
EneInd[b]	-4.199	-4.305	-4.736 (1991M2*; 2004M9*)	-4.831 (1990M11*; 2003M11*)
CPI	-3.461	-3.730	-3.079 (1992M5*; 2007M10*)	-2.570 (1985M11*; 2002M11*)
Price levels				
Oil	-4.731	-4.755	-6.299* (2005M9*; 2015M2*)	-6110* (2003M11*; 2013M5*)
Coal	-5.245* (2006M11)	-5.253* (2006M11)	-2.869 (2007M3*; 2008M1*)	-5.791* (2006M9*; 2007M6*)
Gas[c]	-2.337	-2.454	-2.462 (2004M11*; 2014M5*)	-1.718* (2003M8*; 2013M11*)
Aluminium	-4.532	-4.399	-3.447 (1987M4*; 2004M4*)	-5.603* (2004M8*; 2007M6*)
Copper	-4.471	-4.464	-3.589 (2005M6*; 2014M3*)	-4.131 (2004M4*; 2013M6)
Zinc	-4.236	-4.236	-4.209 (2005M6*; 2006M10*)	-5.614* (2004M10*; 2007M4*)
Nickel	-4.027	-4.626	-4.155 (2005M9*; 2007M1*)	-6.867 (2005M2*; 2006M4*)
Wheat	-3.663	-3.673	-3.747 (2007M4*; 2013M10*)	-5.190 (2009M11*; 2013M1*)
Corn	-4.572	-4.577	-5.119 (2009M11*; 2013M1*)	-5.688* (2009M5*; 2012M6*)
Soy	-5.091* (2006M10)	-5.102* (2007M5)	-4.749 (2007M3*; 2013M11*)	-6.061 (2006M3*; 2013M3*)
Beef	-4.009	-4.574	-4.244 (2009M5*; 2018M8*)	-4.532 (2008M9*; 2018M9*)
Logarithm of price levels				
Oil	-4.188	-4.363	-5.055 (1985M4*; 2003M9*)	-4..888 (1998M1*; 2003M11*)
Coal	-4.273	-4.597	-3.476 (2007M3*; 2008M1*)	-4..753 (2002M9*; 2005M9*)
Gas[c]	-2.598	-4.706	-4.327 (1993M10*; 2003M8*)	-6..451* (1993M11*; 2003M9*)
Aluminium	-4.349	-4.639	-4.366 (1987M4*; 2003M4*)	-5.307* (2004M5*; 2007M6*)
Copper	-4.584	-4.699	-4.392 (1987M1*; 2005M6*)	-4.820* (1985M2*; 2002M8*)
Zinc	-4.701	-4.708	-4.580 (2005M6*; 2007M1*)	-5.333 (1986M8*; 2004M6*)
Nickel	-3.958	-4.204	-4.401 (1987M1*; 2006M3*)	-4.586 (1986M2*; 2002M3*)
Wheat	-3.972	-3.969	-3.456 (2006M10*; 2013M10*)	-4.270 (2004M10*; 2013M4*)
Corn	-4.750	-4.772	-5.037 (2006M3*; 2013M10*)	-4.969 (2005M7*; 2012M4*)
Soy	-5.343* (2006M10)	-5.390* (2006M10)	-4.472 (2007M3*; 2013M11*)	-5.638* (2005M8*; 2013M3*)
Beef	-4.039	-4.886	-4.432 (1993M1*; 2009M5*)	-4.169 (2002M4*; 2008M9*)

*Statistically significant at 5% confidence level.
[a] 1991M1-2021M12.
[b] 1992M1-2021M12.
[c] 1985M1-2021M12.
[e] Zivot Andrews (ZA) unit-root test with one endogenous structural break in the intercept or in both the intercept and the deterministic trend; lags selected with AIC between 6 and 12 months; only statistically significant breaks are reported.
[f] Clemente, Montanes and Reyes (CMR) unit-root test with two endogenous breaks (mean shifts) and deterministic trend; lags selected with AIC between 6 and 12 months; AO=Additive Outlier and IO=Innovational Outlier specifications; only statistically significant breaks are reported.

Table A3. Granger causality test (χ^2) of VAR model estimates with Oil, Coal, Natural Gas and CPI as endogenous variables (1985M1-2021M12)[a,b]

	Price levels	Logarithms of price levels
Crude oil		
Coal	13.51*	8.529*
Gas	3.558	9.221*
CPI	5.138	8.354*
Structural break dummies: 2003M11; 20013M5	4.955*; 1.273	-0.069*; -0.021
Coal		
Crude oil	16.09*	8.6054*
Gas	7.41	0.01853
CPI	3.251	0.77946
Structural break dummies: 2003M11; 20013M5	2.131; -0.375	0.039*; 0.004
Gas		
Crude oil	0.739	1.253
Coal	56.59*	3.353
CPI	5.204	6.060
Structural break dummies: 2003M11; 20013M5	-0.257; -0.052	-0.034; -0.035
CPI		
Crude oil	67.25*	47.25*
Coal	17.18*	15.91*
Gas	5.573	0.876
Structural break dummies: 2003M11; 20013M5	0.067; 0.043	0.001; 0.000

*Statistically significant at 5% confidence level.
[a] The period considered depends on natural gas data availability.
[b] The VAR model specification includes a drift, a deterministic trend and lags decided on the basis of AIC.