Appendices

Appendix A Model description

Demand and supply equations

Aggregate output in industry b (Y_b) is composed of a hypothetical aggregate energy input (AEN_b), aggregate intermediate input (AIN_b) and aggregate factor input (APR_b) according a CES production function with constant returns to scale (see glossary at the end of this Appendix for overview of variables, coefficients and sets). Intermediate inputs g in industry b ($IN_{b,g}$) are transformed into an aggregate energy and aggregate intermediate input according to the CES production functions with constant returns to scale. Labor ($PR_{b,1}$) and capital ($PR_{b,2}$) in industry b are transformed into the aggregate factor input, using a CES production function with constant returns to scale. Cost minimization yields CES demand functions for the aggregate energy (A.1), aggregate intermediate (A.2), aggregate factor (A.3), energy (A.4), intermediate (A.5) and factor inputs (A.6):

$$AEN_b = f^{CES}_{AEN_b}(Y_b, WAEN_b, WAIN_b, WAPR_b) \qquad \forall b \in B \qquad \text{(A.1)}$$

$$AIN_b = f^{CES}_{AIN_b}(Y_b, WAEN_b, WAIN_b, WAPR_b) \qquad \forall b \in B \qquad \text{(A.2)}$$

$$APR_b = f^{CES}_{APR_b}(Y_b, WAEN_b, WAIN_b, WAPR_b) \qquad \forall b \in B \qquad \text{(A.3)}$$

$$IN_{b,g} = f^{CES}_{EN_{b,g}}(AEN_b, \boldsymbol{WIN_g}) \qquad \forall b \in B, \forall g \in S_{en} \qquad \text{(A.4)}$$

$$IN_{b,g} = f^{CES}_{IN_{b,g}}(AIN_b, \boldsymbol{WIN_g}) \qquad \forall b \in B, \forall g \in S_{mat} \qquad \text{(A.5)}$$

$$PR_{b,j} = f^{CES}_{PR_{b,j}}(APR_b, \boldsymbol{WPR_{b,j}}) \qquad \forall b \in B, \forall j \in J \qquad \text{(A.6)}$$

Supply of output g by industry b ($YY_{b,g}$) is proportional to the aggregate output (Y_b) by industry b (A.7). Aggregation of outputs over industries gives domestic production (DP_g) of commodity g (A.8):

$$YY_{b,g} = \delta^Y_{b,g} \times Y_b \quad \sum_{g \in G} \delta^Y_{b,g} = 1 \qquad \forall b \in B, \forall g \in G \qquad \text{(A.7)}$$

$$DP_g = \sum_{b=1}^{B} YY_{b,g} \qquad \forall g \in G \qquad \text{(A.8)}$$

Domestic production (DP_g) and imports (IM_g) are aggregated into total supply of commodity g (SP_g) using a CES production function with constant returns to scale. This implies that the Armington assumption is adopted (see Dervis et al., 1982, p.221). The total supply is then divided into domestic use (DU_g) and exports (EX_g) using a CET product transformation function with constant returns to scale. Cost minimization yields CES demand equations for domestic production (A.9) and imports (A.10) and revenue maximization yields CET supply equations for domestic use (A.11) and exports (A.12):

$$DP_g = f^{CES}_{DP_g}(SP_g, WDP_g, WIM_g) \qquad \forall g \in G \qquad \text{(A.9)}$$

$$IM_g = f^{CES}_{IM_g}(SP_g, WDP_g, WIM_g) \qquad \forall g \in G \qquad \text{(A.10)}$$

$$DU_g = f^{CET}_{DU_g}(SP_g, WDU_g, WEX_g) \qquad \forall g \in G \qquad \text{(A.11)}$$

$$EX_g = f^{CET}_{EX_g}(SP_g, WDU_g, WEX_g) \qquad \forall g \in G \qquad \text{(A.12)}$$

Total labour (j=1) and total capital (j=2) available in the economy (TPR_j) are divided into supply of labour ($PR_{b,1}$) and capital ($PR_{b,2}$) by industry using CET product transformation functions with constant returns to scale. Revenue maximisation yields supply functions for labor and capital, respectively (A.13):

$$PR_{b,j} = f^{CET}_{pr_{b,j}}(\overline{TPR_j}, \boldsymbol{WPR_{b,j}}) \qquad \forall b \in B, \forall j \in J \qquad \text{(A.13)}$$

Maximization of the CES utility functions yields CES demand equations for the private household (A.14) and public household (A.15):

$$CON_g = f^{CES}_{con_g}(EXP^{con}, \boldsymbol{WCON_g}) \qquad \forall g \in G \qquad \text{(A.14)}$$

$$GOV_g = f^{CES}_{gov_g}(EXP^{gov}, \boldsymbol{WGOV_g}) \qquad \forall g \in G \qquad \text{(A.15)}$$

The demand for investment goods (X^{inv}_g) is given by (A.16):

$$INV_g = \delta^{inv}_g \times INV \quad \sum_{g \in g} \delta^{inv}_g = 1 \qquad \forall g \in G \qquad \text{(A.16)}$$

Zero-profit conditions

The value of the disaggregated outputs by industry is equal to the value of the aggregate output produced by industry (A.17). The value of aggregate output equals the value of the aggregate energy input, aggregate intermediate input and aggregate factor input (A.18):

$$\sum_{g \in G} WDP_g \, Y_{b,g} = WY_b \times Y_b \qquad \forall b \in B \qquad \text{(A.17)}$$

$$WY_b Y_b = WAEN_b \times AEN_b + WAIN_b \times AIN_b + WAPR_b \times APR_b \qquad \forall b \in B \qquad \text{(A.18)}$$

The value of the aggregate energy input is equal to the value of the energy inputs by industry (A.19). The value of the aggregate intermediate input equals the value of the intermediate inputs by industry (A.20). The value of the aggregate factor input equals the value of labour and capital by industry (A.21):

$$WAEN_b \times AEN_b = \sum_{g \in S_{en}} WIN_{b,g} \times IN_{b,g} \qquad \forall b \in B \qquad \text{(A.19)}$$

$$WAIN_b \times AIN_b = \sum_{g \in S_{mat}} WIN_{b,g} \times IN_{b,g} \qquad \forall b \in B \qquad \text{(A.20)}$$

$$WNAPR_b \times APR_b = \sum_{j=1}^{2} WPR_{b,j} \times PR_{b,j} \qquad \forall b \in B \qquad \text{(A.21)}$$

The value of total supply (SP_g) equals the sum of the value of domestic production and imports (A.22) and the sum of the value of domestic use and exports by commodity (A.23):

$$WSP_g \times SP_g = WDP_g \times DP_g + WIM_g \times IM_g \qquad \forall g \in G \qquad \text{(A.22)}$$

$$WSP_g \times SP_g = WDU_g \times DU_g + WEX_g \times EX_g \qquad \forall g \in G \qquad \text{(A.23)}$$

The value of the supply of labour and capital equals the value of the total availability of labour and capital, respectively (A.24).

$$\sum_{B \in b} \left(WPR_{b,j} \times PR_{b,j} \right) = WTPR_j \times \overline{TPR_j} \qquad \forall j \in J \qquad \text{(A.24)}$$

The value of the demand for individual investment goods equals the expenditure on investment (A.25):

$$\sum_{g \in G}\left(WINV_g \times INV_g\right) = WINV \times INV \qquad \text{(A.25)}$$

Margins

The value of demand/supply for margins per commodity (MAR_g) is assumed to form a share of the value of the different demand categories and is given by (A.26).

$$MAR_g = m_g^{sp} \times WDU_g \times DU_g + m_g^{sp} \times WEX_g \times EX_g \qquad \forall g \in G \qquad \text{(A.26)}$$

The total value of the margins is zero (A.27):

$$\sum_{g \in G} MAR_g = 0 \qquad \text{(A.27)}$$

Equilibrium conditions for commodities

Total domestic use equals intermediate, private household, public household and investment demand (A.28):

$$DU_g = \sum_{b \in B} IN_{b,g} + CON_g + GOV_g + INV_g \qquad \forall g \in G \qquad \text{(A.28)}$$

Price equations

Indirect taxes and margins drive a wedge between the demanders' and suppliers' price of domestic use. This applies to commodities consumed by the private household (A.29), the public household (A.30), as investment commodities (A.31), as intermediate inputs (A.32) and as exports (A.33):

$$WCON_g = \left(1 + m_g^{sp} + t_g^{sp}\right) \times WDU_g \quad \text{(A.29)}$$

$$WGOV_g = \left(1 + m_g^{sp} + t_g^{sp}\right) \times WDU_g \quad \text{(A.30)}$$

$$WINV_g = \left(1 + m_g^{sp} + t_g^{sp}\right) \times WDU_g \quad \text{(A.31)}$$

$$WIN_g = \left(1 + m_g^{sp} + t_g^{sp}\right) \times WDU_g \quad \text{(A.32)}$$

$$WGEX_g = \left(1 + m_g^{sp} + t_g^{sp}\right) \times WEX_g \quad \text{(A.33)}$$

The price received for exports (A.34) and paid for imports (A.35) are equal to the world market price times the exchange rate:

$$WGEX_g = \overline{WPEX_g} \times \overline{ER} \quad \text{(A.34)}$$

$$WIM_g = \overline{WPIM_g} \times \overline{ER} \quad \text{(A.35)}$$

Non-product related taxes are levied on the total value of labour and capital used by industry (A.36):

$$WAPR_b = \left(1 + t_b^{apr}\right) \times WNAPR_b \quad \text{(A.36)}$$

Income formation and distribution

Tax revenue from product (PTX) and non-product related taxes ($NPTX$) and social contribution ($SOCCON$) is given by equations A.37, A.38 and A.39, respectively:

$$PTX = \sum_{g \in S}\left(t_g^{sp} \times WDU_g \times DU_g + t_g^{sp} \times WEX_g \times EX_g\right) \quad \text{(A.37)}$$

$$NPTX = \sum_{b \in B}\left(t_b^{apr} \times WNAPR_b \times APR_b\right) \quad \text{(A.38)}$$

$$SOCCON = r_{soccon} \times LABI \quad \text{(A.39)}$$

Labour income ($LABI$), capital income ($CAPI$) and capital depreciation (DEP) are given by equation A.40, A.41 and A.42 respectively:

$$LABI = WTPR_1 \times \overline{TPR_1} \quad \text{(A.40)}$$

$$CAPI = WTPR_2 \times \overline{TPR_2} \quad \text{(A.41)}$$

$$DEP = r_{capdep} \times CAPI \quad \text{(A.42)}$$

The income (I^{con}), expenditure EXP^{con} and savings (SAV^{con}) of the private household are given by equations A.43, A.44 and A.45 respectively:

$$I^{con} = (1 - r_{soccon}) \times LABI + (1 - r_{capdep}) \times CAPI \quad \text{(A.43)}$$

$$EXP^{con} = (1 - s^{con}) \times I^{con} \quad \text{(A.44)}$$

$$SAV^{con} = s^{con} \times I^{con} \quad \text{(A.45)}$$

The income (I^{gov}), expenditure (EXP^{gov}) and savings (SAV^{gov}) of the public household are given by equations A.46, A.47 and A.48, respectively:

$$I^{gov} = PTX + NPTX + SOCCON \tag{A.46}$$

$$EXP^{gov} = (1 - s^{gov}) \times I^{gov} \tag{A.47}$$

$$SAV^{gov} = s^{gov} \times I^{gov} \tag{A.48}$$

Total savings (A.49) equal the sum of the savings of the private household, public household and capital depreciation minus the surplus on the balance of trade (BBAR). As the balance of trade is expressed in foreign price it must be multiplied by the exchange rate (ER):

$$SAV = SAV^{con} + SAV^{gov} + DEP - BBAR \times \overline{ER} \tag{A.49}$$

The surplus on the balance of trade (A.50) equals:

$$BBAR = \sum_{g \in G}\left(\overline{WPEX}_g \times EX_g\right) - \sum_{g \in G}\left(\overline{WPIM}_g \times IM_g\right) \tag{A.50}$$

The value of investment ($INVT$) equals the value of the savings (A.51) but also the value of the investment commodities (A.52). Due to Walras' law, one equation must be dropped to keep the model identified. In the operational model this is Equation A.52.

$$INVT = SAV \tag{A.51}$$

$$INVT = WINV \times INV \tag{A.52}$$

Price numéraire

An applied general equilibrium model is homogenous of degree zero. This model selects the exchange rate as price numéraire.

Environment

CO_2, CH_4, N_2O emissions of an industry ($EMIS_b$) equals the share (amount) of emissions (δ_b^{emis}) emitted by the industry($Y_{b\ old}$).

$$\delta_b^{emis} = EMIS_{b\ old}/Y_{b\ old} \tag{A.53}$$

$$EMIS_b = \delta_b^{emis} \times Y_b \tag{A.54}$$

Welfare change

As a welfare measure, the Laspeyres index of real income change is taken. This index compares commodity bundles between two equilibria (e.g. before and after a policy change), using the prices of the initial equilibrium (A.55). This welfare measure allows for the calculation of the welfare effects of savings other than the private savings of which the underlying optimizing behaviour is not modelled explicitly (i.e., capital depreciation, government deficit and the balance of trade). Since savings are equal to investments, the bundle of investment commodities represent welfare derived from saving.

$$WELF =$$
$$\sum_{g\in S}\left(WCON_g^{old} \times CON_g^{old}\right)+\sum_{g\in S}\left(WGOV_g^{old} \times GOV_g^{old}\right) + \sum_{g\in S}\left(WINV_g^{old} \times INV_g^{old}\right) -$$
$$\sum_{g\in S}\left(WCON_g^{old} \times CON_g\ \right) - \sum_{g\in S}\left(WGOV_g^{old} \times GOV_g\ \right) - \sum_{g\in S}\left(WINV_g^{old} \times INV_g\ \right) \tag{A.55}$$

Glossary

Variables:

Quantities

Y_b	aggregate output of industry b
AEN_b	aggregate intermediate energy input in industry b
AIN_b	aggregate intermediate materials input in industry b
APR_b	aggregate factor input in industry b
$IN_{b,g}$	intermediate input g in industry b
$PR_{b,j}$	labour (j=1) and capital (j=2) in industry b
$YY_{b,g}$	output g of industry b
DP_g	domestic production of commodity g
IM_g	import of commodity g
SP_g	total supply of commodity g
DU_g	domestic use of commodity g
EX_g	export of commodity g
$\overline{TPR_J}$	labour (j=1) and capital (j=2) endowment in the economy
CON_g	consumer demand for commodity g
GOV_g	government demand for commodity g
INV_g	demand of investment good g
INV	aggregate investment good
$EMIS_b$	emissions of industry b

Prices (quantity symbols plus a W)

$WAEN_b$	price of aggregate intermediate energy input in industry b
$WAIN_b$	price of aggregate intermediate materials input in industry b

$WAPR_b$ price of aggregate factor input including tax in industry b

$WNAPR_b$ price of aggregate factor input excluding the tax in industry b

WIN_g price of intermediate input g

$WPR_{b,j}$ price of labour (j=1) or capital (j=2) used in industry b

WIM_g price of import of commodity g

$\overline{WPIM_g}$ world market price of import of commodity g

WDP_g price of domestic production of commodity g

WY_b price of aggregate output of industry b

WSP_g price of total supply of commodity g

WDU_g price of domestic use of commodity g

WEX_g price of export excluding taxes and margins of commodity g

$WGEX_g$ price of export including taxes and margins of commodity g

$\overline{WPEX_g}$ world market price of export of commodity g

$WCON_g$ price of commodity g demanded by the private household

$WGOV_g$ price of commodity g demanded by the public household

$WTPR_j$ price of labour (j=1) and capital (j=2) endowment in the economy

$WINV_g$ price of investment good g

$WINV$ price of aggregate investment good

Other

$\overline{ER}$ exchange rate

MAR_g value of demand/supply of margins by commodity g

$LABI$ labour income

$CAPI$ capital income

DEP capital depreciation

I^{con}	income of private household
I^{gov}	income of public household
EXP^{con}	expenditure of private household
EXP^{gov}	expenditure of public household
SAV^{con}	expenditure of private household
SAV^{gov}	expenditure of public household
SAV	total savings
$BBAR$	surplus on the trade balance (in foreign prices)
DEP	capital depreciation
$INVT$	value of the investment
PTX	value of the product related taxes
$NPTX$	value of the non-product related taxes
$SOCCON$	social contributions
$WELF$	welfare change

Fixed coefficients

δ_b^{emis}	coefficient dividing aggregate emissions by industry b
m_g^{sp}	margins for commodity g
t_g^{sp}	tax rate for product related taxes on commodity g
t_b^{apr}	tax rate for non-product related taxes in industry b
$\delta_{b,g}^{Y}$	coefficient dividing aggregate output into outputs g in industry b
δ_g^{inv}	coefficient dividing aggregate investment good into investment good g
s^{con}	savings rate of the private household
s^{gov}	savings rate of the public household
r_{soccon}	share of labour income that goes to social contributions

r_{capdep} capital depreciation as share of capital income

Sets and subsets:

G: goods, g = 1 to 60

B: industries, b = 1 to 60

J: factors, j = 1 (labour) j = 2 (capital)

$S_{en} \subset G$: subset energy commodities: g = 24,25

$S_{mat} \subset G$: subset materials: g = 1,…,23,26,…,60

Miscellaneous

gov = public household; con = private household; inv = investment; en = energy; old = base year value; CES = Constant Elasticity Substitution; CET = Constant Elasticity of Transformation

Bold printed variables represent a vector; variables with a bar represent exogenous variables.

Table A1 Aggregation level of commodities and industries in Supply-Use tables

CPA/NACE code	**Commodity/Industry**
A01	Crop and animal production, hunting and related service activities
A02	Forestry and logging
A03	Fishing and aquaculture
B	Mining and quarrying
C10_12	Manufacture of food products, beverages and tobacco products
C13_15	Manufacture of textiles, wearing apparel and leather products

C16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
C17	Manufacture of paper and paper products
C18	Printing and reproduction of recorded media
C19	Manufacture of coke and refined petroleum products
C20	Manufacture of chemicals and chemical products
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
C22	Manufacture of rubber and plastic products
C23	Manufacture of other non-metallic mineral products
C24	Manufacture of basic metals
C25	Manufacture of fabricated metal products, except machinery and equipment
C26	Manufacture of computer, electronic and optical products
C27	Manufacture of electrical equipment
C28	Manufacture of machinery and equipment n.e.c.
C29	Manufacture of motor vehicles, trailers and semi-trailers
C30	Manufacture of other transport equipment
C31_32	Manufacture of furniture; other manufacturing
C33	Repair and installation of machinery and equipment
D35-1	Biobased energy
D35-2	Non-biobased energy
E36	Water collection, treatment and supply
E37_39	Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
F	Construction

G	Wholesale and retail trade
H49	Land transport and transport via pipelines
H50_51	Water and Air transport
H52	Warehousing and support activities for transportation
H53	Postal and courier activities
I	Accommodation and food service activities
J58	Publishing activities
J59_60	Motion picture, video and television program production, sound recording and music publishing activities; programming and broadcasting activities
J61	Telecommunications
J62_63	Computer programming, consultancy and related activities; information service activities
K64	Financial service activities, except insurance and pension funding
K65	Insurance, reinsurance and pension funding, except compulsory social security
K66	Activities auxiliary to financial services and insurance activities
L68	Real estate activities
M69_70	Legal and accounting activities; activities of head offices; management consultancy activities
M71	Architectural and engineering activities; technical testing and analysis
M72	Scientific research and development
M73	Advertising and market research
M74_75	Other professional, scientific and technical activities; veterinary activities
N77	Rental and leasing activities
N78	Employment activities

N79	Travel agency, tour operator reservation service and related activities
N80_82	Security and investigation activities; services to buildings and landscape activities; office administrative, office support and other business support activities
O	Public administration and defense; compulsory social security
P	Education
Q86	Human health activities
Q87_88	Social work activities
R90_92	Creative, arts and entertainment activities; libraries, archives, museums and other cultural activities; gambling and betting activities
R93	Sports activities and amusement and recreation activities
S94	Activities of membership organizations
S95	Repair of computers and personal and household goods
S96_T	Other personal service activities

Source: CSB Latvia, 2016

Table A2 Industries ranked according to the largest shares in supply and use of biobased energy commodities

Suppliers of Energy commodities*	**Users of Energy commodities****
Forestry and logging	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
Fishing and aquaculture	Manufacture of chemicals and chemical products

Manufacture of machinery and equipment n.e.c.	Residential care activities and social work activities without accommodation
Rental and leasing activities	Sewerage, waste management, remediation activities
Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials	
Manufacture of chemicals and chemical products	
Manufacture of furniture; other manufacturing	
Retail trade, except of motor vehicles and motorcycles	

* 90%<

**30%<

Source: author`s calculations based on Eurostat, 2021

Table A3 Substitution (sigma) and transformation (omega) elasticities

Symbol	**Function**	**Value**
SIGMAY (B)	CES for output	0.4
SIGMAPR (B)	CES for aggregate primary input	0.4
SIGMAE (B)	CES for aggregate energy input	1.5
SIGMAX (B)	CES for aggregate intermediate input	0.4
SIGMADOMS (G)	CES for domestic supply	1.5

OMEGADOMD (G)	CET for domestic demand	-1.5

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