



Citation: d'Amato, M., & Bambagioni, G. (2023). Discounted Cash Flow Analysis and Prudential Value DCFA Formula. *Aestimum* 83: 59-68. doi: 10.36253/ aestim-14037

Received: December 5, 2022

Accepted: January 24, 2024

Published: April 22, 2024

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Data Availability Statement: The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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Discounted Cash Flow Analysis and Prudential Value DCFA Formula

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Abstract. According to IVS 2023 the terminal value of a DCFA can be calculated in respect of the three fundamental appraisal approach: market, income and cost. Specifically, "Where the asset is expected to continue beyond the explicit forecast period, valuers must estimate the value of the asset at the end of that period. The terminal value is then discounted back to the valuation date, normally using the same discount rate as applied to the forecast cash flow" (IVS 105 Valuation Approaches and Methods, para 50.20). Although academic and professional normally refer to direct capitalization to calculate the exit value (scrap value, going out value) different approaches are possible. In particular it is possible to calculate the exit value using the market approach. The present work starts from an original model of Discounted Cash Flow Analysis proposed by Simonotti (2006) to propose possible alternative to the original formula. The model proposed, as defined for the paper "Prudential Value DCFA Formula (PVF)" provides methodological alternatives to the original model in order to apply the method in different property market conditions allowing the appraiser to represent even property market in recovery or falling market phase, or prudential assessment of worth and opinion of value. In particular the proposed model replace the compound growth of the property value included in the traditional Simonotti's model, with a linear growth or even a combination of both. The paper emphasize how property valuation standards can not be considered as immutable but they may be considered in constant evolution. In delivering an opinion of value for mortgage lending purposes the approach of professional academician and real estate analyst need to be able to respond to change of external conditions, in consideration of the sustainability of the cash flows in the long term, for the entire duration of the loan (and therefore for the purpose of identifying the long-term sustainable value of the asset). The approach is functional, inter alia, for the purpose of identifying the "prudential value" as defined by the innovative banking regulation (Basel 3) according to the definition of loan-to-value (LTV) ratio for loan origination and monitoring.

Keywords: Discounted Cash Flow Analysis (DCFA), IVS, Prudential Value, Property Valuation, Simonotti DCFA Model, Long-Term Sustainable Value, Loanto-Value (LTV) ratio, Sustainable investment, Sustainable lending.
 JEL code: C19, E32, E43, R39.

1. INTRODUCTION

Discounted cash Flow Analysis (DCFA) is a model for the valuation of both investment value (assessment of worth) and market value of income producing properties. In addition to the investment value, the methods are applied to the study of atypical cash flows of properties, in particular situations not attributable to the direct or financial schemes of the other two methods of capitalization of income (Direct capitalization and Yield capitalization), generally consisting of properties subject to transformation or development that provide for exits and revenues over a period of time connected to the construction time of the construction site and marketing (or management) of the construction work carried out or redeveloped¹. This family of models is recalled in the International Valuation Standards (IVSC, 2022, IVS 105, Valuation Approaches and Methods para 50.20 and followings) and in the Italian national standards Codice delle Valutazioni Immobiliari (Chap. 8 para 3.2.3). The methodology was also implemented in the "Linee Guide per la valutazione degli immobili a garanzia delle esposizioni creditizie" (i.e. Guidelines for the valuation of properties as collateral for credit exposures, promoted by ABI and others Italian organizations) (Chapter 8 para 2.5.1). Discounted Cash Flow Analysis is not a homogeneous group of property valuation techniques. The method is often adopted, with different formulas, in Commonwealth context, and other models normally are applied in US. In Italy, normally, are applied both models according to the valuer background. In the IVS 2022 the "terminal value" of a DCFA can be calculated in respect of the three fundamental appraisal approach: market, income, and cost. Specifically, "Where the asset is expected to continue beyond the explicit forecast period, valuers must estimate the value of the asset at the end of that period. The terminal value is then discounted back to the valuation date, normally using the same discount rate as applied to the forecast cash flow" (IVS 105 Valuation Approaches and Methods, para 50.20). Among the others Italian models, we consider the so called Simonotti Model of Discounted Cash Flow Analysis (Simonotti, 2006). In this model any prospective analysis should consider the integration in the analysis

itself of the recurring changes of the property market conditions, in consideration of the cyclical nature of the markets in the long term, as well as the sustainability of the long-term value in consideration of the characteristics of the asset and the assessment of its life cycle. The original model is based on a hypothesis of compound growth of the value of the property at rate d. This term is different from the well-known g-factor, a term used both in direct capitalization with explicit growth and in terminal value calculation in DCF. The term g-factor indicates a growth both in term of rent and in term of property value, in the original Simonotti's model the term d refers to the growth of property price only. In both cases the growth is supposed to be a compound growth. A question may be raised: is this the only method to represent real estate growth? In general term real estate market is cyclical. This is particularly true in specific market condition (property market with a low growth or even falling market) and above all in the determination of mortgage lending value that should represent "a long-term, sustainable value as a stable basis for judging the suitability of a property as a security for a mortgage which will continue through potential market fluctuations" (European Banking Authority, 2015). According to Quentin (2009), mortgage lending value calculation should be: "unattached from temporary, e.g. economically induced, fluctuations in value on the relevant property market and excluding speculative elements". This is particularly true after the Global Financial Crisis. The seminal report of the Long-Term Working Group of the Property Industry Alliance Debt Group (2017) stated that "...they must put in place and hardwire all the tools needed to identify when the market might be overheating and have a regular (likely quarterly) system that monitors that probability. This is where this long-term value methodology paper comes in..." (Foreword, Long-Term Working Group of the Property Industry Alliance Debt Group, 2020, p.1). They presented three possible solutions for the valuation of longterm sustainable value: the Adjusted Market Value, the Investment Value based on a Discounted Cash Flow Analysis, Mortgage Lending Value based on German "Beleihungswert". The first one has been proposed in the document as the most reliable method, consisting in a market value adjusted by a factor calculated on the difference between market value and long-term capital values. It is an empirical solution that may have a wide application among professional property valuers. A further solution is based on the calculation of the investment value using a Discounted Cash Flow modelling as previously indicated in academic literature (Crosby, 2021). The final solution "Beleihungswert" is the defini-

¹ See the Italian Codice delle Valutazioni Immobiliari (Chap 8, para 3.2.3): "L'analisi del flusso di cassa scontato (discounted cash flow analysis) si applica allo studio dei flussi di cassa atipici di immobili in situazioni particolari non riconducibili agli schemi diretti o finanziari degli altri due metodi della capitalizzazione del reddito, in genere costituiti dagli immobili oggetto di trasformazione o sviluppo che prevedono uscite e ricavi in un arco temporale connesso ai tempi di realizzazione del cantiere e commercializzazione (o gestione) dell'opera edilizia realizzata o riqualificata" (Simonotti and Bambagioni, 2018).

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tion of the opinion of value of the asset with the highest degree of probability over the period of loan (Werth, 1998; Gondring & Lorenz, 2001; Ruchardt, 2001).A second report proposed by the same group on 2020 explored the "...models for estimates of longer run capitalisation (cap) rates ..." (IPF Research Program, Longterm Value Methodologies in Commercial real Estate Landing, p. 4). The proposed model of mortgage lending value by Pfandbrief Act (www.pfandbrief.de/site/en/vdp/ real_estate/valuation/mortgage_lending_value.html) originated a debate (Crosby et al., 2011) based on the nature under the cycle or through the cycle meaning of mortgage lending value. According to the former approach mortgage lending value should be approximated to a straight line under the market level conditioned by the effect of the market cycle. In the latter approach the mortgage lending value should be approximated to market value. The difference is related to Institutional contexts, too. In UK there was not a specific definition of mortgage lending value. On the other side German Pfandbrief Act originated the previously described ridiculous former approach. Bienert and Brunauer (2017) showed the importance of deriving mortgage lending value from market value avoiding a lump sum estimation. Simonotti's DCF model is normally used for mortgage lending value determination of income producing properties. The proposed variant to the original Simonotti's DCF model tries to reach prudent opinion of value following previous contributions in the literature (Nordlund, 2008; Crosby and Hordijk, 2021) in order to determine a value "through the cycle". In Italian literature a further contribution for mortgage lending value determination is provided by the application of VaR methodology (Tajani and Morano, 2017). This paper proposes a variation of the original Simonotti's formula in order to allow the appraiser, investor, banks, and lenders of real estate development projects to consider also specific critical market phases with a prudent assessment of the value. In this variant of the original model, we focus on the term d that indicates the growth in capital term of the value over the time. It is useful to stress that in Simonotti's model the term d is distinct by the common term g defined growth factor recurring in academic literature (Gordon, 1958; Gordon and Shapiro, 1962). Whilst the term g is referred to the growth both in term of price and in term of rent in Simonotti's model the term d is only referred to property price growth. Therefore, d can be considered a part of the well-known g-factor. A prudential approach is relevant both for mitigating the risks associated with the investment and for the purposes of loan origination and monitoring. In fact, the banking regulations of reference are increasingly oriented towards the definition of the loan-to-value (LTV) ratio on the basis of a prudential value. Prudential Value is defined in Basel III (2017) as: "Value of the property: the valuation must be appraised independently using prudently conservative valuation criteria. To ensure that the value of the property is appraised in a prudently conservative manner, the valuation must exclude expectations of price increases and must be adjusted to take into account the potential for the current market price to be significantly above the value that would be sustainable over the life of the loan. National supervisors should provide guidance, setting out prudent valuation criteria where such guidance does not already exist under national law. If a market value can be determined, the valuation should not be higher than the market value [...] In the case where the mortgage loan is financing the purchase of the property, the value of the property for LTV purposes will not be higher than the effective purchase price"². It is worth to notice that excluding expectation of price increases should not be interpreted literally. It is possible to assume price increase without speculative component. It is easy to observe how the models proposed by the Long-Term Working Group include property price growth term. The phrase is referred to the need of adjusting the market value avoiding speculative components. (Long-Term Working Group of the Property Industry Alliance Debt Group,2020, p.12). The contribution is organized as follows: the following paragraph introduce the methodology of Discounted Cash Flow Analysis in general terms whilst in the following paragraph there is an introduction to Simonotti DCFA model and the proposed Prudential Value DCFA formula (PVF). Final remarks and future directions of research will be offered at the end.

2. DISCOUNTED CASH FLOW ANALYSIS AND SUSTAINABLE LENDING

The origin of Discounted Cash Flow Analysis as a valuation procedure is not academic. For the first time was an important academic of last century who discovered the use of such technique among real estate professionals (Graaskamp, 1969). In 1976 an analysis on 158 corporation working in the sector of real estate showed how they used the before tax cash flow modelling for 60% and after-tax models for 22% (Wiley, 1976). In another work (Farragher, 1982), 354 real estate investment companies were discovered to use for 66% the Discounted Cash Flow Analysis both in before

² See: BIS, Basel Committee on Banking Supervision – Basel III: Finalising post-crisis reforms, https://www.bis.org/bcbs/publ/d424.pdf

tax and in after tax. Growing interest was registered in further works (Page, 1983; Webb, 1984). A further enquiry (McIntosh, 1986) discovered that among 32 managers from the most important real estate investment companies in US the greatest part preferred using Discounted Cash Flow Analysis instead of the traditional Direct Capitalization. Discounted Cash flow analysis was introduced in UK by the seminal paper of Marshall (Marshall, 1976). Studies on the relationship between inputs and outputs of DCFA have been more frequent between 1960 and 1980. (Downs, 1966; Ratcliff, 1972; Dilmore, 1971). Several reasons may be in favour of the use of Discounted Cash Flow. Several contributions highlighted the role of uncertainty and the forecast in Discounted Cash Flow Analysis (French Cooper, 2000). Reliability in the valuation of cash flow has been required by (Willinson, 1999) and vacancy rate and market analysis (Rabianski, 2002). Critics have been raised in the application of Discounted Cash Flow Analysis which "...can be very accurate, but it can also be very inaccurate, and the degree of accuracy will depend upon the accuracy of the valuation inputs ..." (Millington, 2000, p.187); an analysis that, as authors of this publication, we fully share. A reliable forecast of inputs in the valuation has been underlined in further studies. "... The importance of achieving rental growth may be judged from the knowledge that inflation has risen again recently and that the average annual rate of inflation in the 64 years from 1925 has been 5,2% ..." (Scarrett, 2000). Further contributions in the same contexts (Hendershott and Hendershott, 2002; Taylor and Rubin, 2002; Wheaton et al., 2001). A warning was launched on the application of Discounted Cash Flow Analysis in the construction of real estate index (Hordijk and Van de Ridder, 2004). DCFA can be also a tool for assessment of worth or investment value determination. In methodological terms the model assumes a holding period of the property followed by the direct capitalization using a going out cap rate to determine the exit value (going out value, exit value, scrap value). The valuation method is useful both for market value and investment value (assessment of worth) determination. US DCFA models are normally focused on the role of taxation. We have therefore before tax and after-tax modelling (Lusht, 1997). The going out value is normally calculated using a dividend discount model or a normal capitalization. In the UK, it is possible to see short cut DCFA, hybrid, and equated yield models (Millington, 2000). DCF is also used for property valuation based on mortgage lending value. The sustainability of the value of the property, subject to the mortgage guarantee (collateral), is an essential factor in identifying the degree of risk of the loan. At the same time, projections based on historical series and statistical datasets, in an increasingly dynamic, global, and interconnected markets, cannot be considered reliable for a very long-time horizon. In the case of analyzes related to very long-term mortgages, the situation could be affected by a radical change to an existing industry or market due to factors of significant impact and discontinuity. Disruptions, such as those experencied during the global financial crisis (originating in 2008), the Covid-19 pandemic, and regulatory innovations such as those that could be introduced by the Basel 3 Agreement and/or by other Supervisors, requires a growing attention to (i) the quality of the data, (ii) the skills of the valuers, and (iii) the valuation methods since some approaches, more than others, are suitable for mitigating the effects of the estimate (sometimes an overestimation) of the market value defined in the loan origination phase (d'Amato et al., 2023).

3. SUSTAINABLE INVESTMENTS AND PRUDENTIAL VALUE DCFA FORMULA (PVF)

Within the Italian context, while taking into account the IVS, Simonotti (Simonotti, 2006) proposed a different approach to Discounted Cash Flow Analysis. In compliance with to IVS 1997, Simonotti proposed the valuation of exit value, scrap value, going out value using market value. The original idea was the computation of the terminal value using a market-oriented approach as follows:

$$V = \sum_{t=1}^{n} \frac{F_t}{(1+i)^t} + \frac{V(1\pm d)^n}{(1+i)^n}$$
(1)

In Formula 1 the value is equal to the sum of the cash flows deriving from the holding period summed up to a terminal value that is the original value to be estimated increased or decreased at a compound growth factor d. It is possible to rewrite the formula as follows:

$$V - \frac{V(1 \pm d)^n}{(1+i)^n} = \sum_{t=1}^n \frac{F_t}{(1+i)^t}$$
(2)

And finally in the Formula 3:

$$V = \frac{\sum_{t=1}^{n} \frac{F_{t}}{(1+i)^{t}}}{1 - \frac{(1\pm d)^{n}}{(1+i)^{n}}}$$
(3)

In the Formula 3: V is the market value (or assessment of worth), the term i is the discount rate, target rate of return, equated yield, the term d is a growth (or decrease) factor in term of capital from the moment of the valuation to the end of the holding period. The model diminishes the number of the inputs normally required for a Discounted Cash Flow Analysis. An important assumption is the compound growth of the price of property at an annual d rate from the moment of the valuation to the end of the holding period. It is worth to say that if d is equal to i the formula is meaningless, like in the original formula of Dividend Discount Model. Generally speaking, the term d is not a financial information that should calculated according to rule of financial maths but a real estate one. It is the perspective real estate growth of property price in the specific period of time, in the specific market segment. In mathematical term, the Simonotti's formula is based on an exponential measure of the growth of the same asset in two different moments. The origin is indicated in the Formula 4 below:

$$P_t = P_0 (1+d)^n \Longrightarrow d = \sqrt[n]{\frac{P_t}{P_0}} -1$$
(4)

The d factor can be positive as negative according to the specific market conditions of market segment. This is related to the specific temporal and statistical trend of property market price. If d is a perspective growth of real estate price, it can be measured in several ways. Although the variation along the time may be positive or negative, the form of the growth may be also linear instead of exponential. In this case the measure of variation can be expressed in the Formula 5 below:

$$P_t = P_0(1+dn) \Longrightarrow d = \frac{P_t - P_0}{P_0 n}$$
(5)

Assuming a linear variation, less intense that exponential one, the original DCFA Simonotti's formula 1 will be changed as follows:

$$V = \sum_{t=1}^{n} \frac{F_t}{(1+i)^t} + \frac{V(1 \pm dn)}{(1+i)^n}$$
(6)

The Formula 6 may be referred to a real estate market in a critical condition. Therefore, the final Formula will be modified as follows:

Table 1 Assumptions for the Determination of Property ValueUsing DCFA.

	Revenue	Revenue Growth	Cost	Cost Growth	Difference between Revenue and Cost
1	1,000.00€	0.01	400.00€	0.015	600.00€
2	1,010.00€		406.00€		604.00€
3	1,020.10€		412.09€		608.01€
4	1,030.30€		418.27€		612.02€
5	1,040.60€		424.54€		616.05€

Table 2 Present Value Calculated on a Discount Rate Varying from0.03 to 0.15.

Discount Rate, Yield, Target Rate of Return,Saggio di Capitalizzazione nella Capitalizzazione Finanziaria	Present Value of Difference between Revenue and Cost
0.03	2,783.46€
0.04	2,705.39€
0.05	2,630.71€
0.06	2,559.23€
0.07	2,490.77€
0.08	2,425.18€
0.09	2,362.30€
0.1	2,301.98€
0.11	2,244.09€
0.12	2,188.51€
0.13	2,135.11€
0.14	2,083.79€
0.15	2,034.44€

$$V = \frac{\sum_{i=1}^{n} \frac{F_{i}}{(1+i)^{i}}}{1 - \frac{(1 \pm dn)}{(1+i)^{n}}}$$
(7)

In Formula 7 V is the value, i is the discount rate, d is the linear growing factor whilst n is the length of the forecast period (also holding period). For the present article the formula will be defined as Prudential Value DCFA Formula (PVF)³. A comparison between the two formulas provides an idea of the impact of the difference. Assuming the valuation of an income producing properties with the following characteristics:

Therefore, the value of the cash flow actualized at the following discount rate, target rate of return, yield rate is indicated in the following Table 2.

³ Or even: "Prudential Value DCFA D'Amato-Bambagioni Formula"

	Discount Rate, Yield, Target Rate of Return,Saggio di Capitalizzazione nella Capitalizzazione Finanziaria	d	Numerator	Denominator	Appraised Value Using Simonotti's DCF Formula	Appraised Value using PVF Formula
1	0.03	0.005	2783.46544	0.115609262	24076.49163	24031.43979
2	0.04	0.005	2705.394469	0.157318203	17196.95753	17174.41266
3	0.05	0.005	2630.712607	0.196688815	13374.99844	13361.62495
4	0.06	0.005	2559.2321295	0.233872621	10942.84619	10934.06841
5	0.07	0.005	2490.777677	0.269010026	9259.051472	9252.889758
6	0.08	0.005	2425.1853319	0.3022312	8024.271274	8019.733808
7	0.09	0.005	2362.3017564	0.333657031	7080.029886	7076.566496
8	0.1	0.005	2301.9834119	0.363399635	6334.578209	6331.859925
9	0.11	0.005	2244.0958377	0.391563282	5731.119181	5728.937614
10	0.12	0.005	2188.5129918	0.418244905	5232.611240	5230.828200
11	0.13	0.005	2135.1166442	0.443534695	4813.866121	4812.386494
12	0.14	0.005	2083.7958200	0.467516626	4457.158749	4455.915016
13	0.15	0.005	2034.4462871	0.490268929	4149.653723	4148.596689

Table 3 Comparing Final Result of Simonotti's DCFA Formula with Prudential Value DCFA Formula.

Finally, it is possible to compare the value using the Simonotti's DCFA and the Prudential Value DCFA Formula (PVF) (Table 3).

The Table 3 indicates a comparison between the opinion of value derived by the two procedures. It is possible to observe the valuation variation between the models assuming the following variation rate between the two results, using the following equation:

$$\Delta = \frac{V_{SM} - V_{PVF}}{V_{PVF}} \tag{8}$$

In the Formula 8 the variation ratio Δ between Simonotti's formula and Prudential Value DCFA formula is calculated as follows: V_{SM} stands for the value calculated using traditional Simonotti model, V_{PVF} stands for the value provided with the same assumptions by the Prudential Value DCFA model. In the following table (Table 4) the variation ratio has been calculated applying both the Simonotti's formula and the Prudential Value DCFA formula for a holding period of five years.

In the Table 4 the column reports the estimation of valuation variation between the two models (Simonotti's

Table 4. Valuation variation. Comparing Final Result of Simonotti's DCFA Formula with Prudential Value DCFA Formula assuming a holding period of 5 years.

Discount	Δ calculation per each value of d holding period 5 years d term					
Rate —	0.005	0.01	0.015	0.02	0.025	
0.03	0.001874704	0.0093295081	0.0278570791	0.073936607	0.22996643	
0.04	0.0013127009	0.006097758	0.0163881892	0.036250573	0.0751743355	
0.05	0.001000887	0.0044837009	0.0114775472	0.023697939	0.0442332071	
0.06	0.00080279150	0.0035166974	0.00875293212	0.0174285469	0.030984888	
0.07	0.00066592316	0.0028732015	0.00702191977	0.0136723641	0.023634025	
0.08	0.00056578752	0.0024145507	0.00582588551	0.0111727136	0.018963705	
).09	0.00048941661	0.0020714136	0.0049508044	0.0093910194	0.015736653	
).1	0.0004293026	0.00180527290	0.0042833693	0.00805799354	0.013375472	
0.11	0.00038079796	0.0015930180	0.0037579812	0.0070240314	0.011574484	
0.12	0.000340871	0.0014199426	0.0033340315	0.0061993706	0.0101567172	
0.13	0.00030746215	0.0012762421	0.00298503610	0.0055268897	0.00901261582	
).14	0.00027911948	0.0011551290	0.00269298613	0.0049685076	0.0080707323	
).15	0.00025479324	0.0010517552	0.00244520871	0.0044978611	0.0072824823	

 Table 5. Valuation variation. Statistical Data for 5 years holding period.

Arithmetic Mean	0.103831988
Maximum Value	0.229966438
Minimum Value	0.0362505738
Dev. Standard	0.0860080264

DCF and PVF) according to different combination of discount rate and progression rate d. The row indicates the estimation of the discount rate whilst the column indicates the progression rate d. Per each cell there is the relative estimation of Δ or valuation variation between the models. It is clear that in every cell the valuation variation is positive therefore the Simonotti's DCF formula is always higher than the PVF as expected not different. It is possible to observe significative difference in term of valuation variation with low discount rate included between 0.03 and 0.04 for a d factor varying between 0.02 and 0.025. In the Table 5 below it is possible to observe the statistical data about the valuation variation between the two different models for a holding period of 5 years.

It is possible to observe that for the interval selected the valuation variation become meaningful reaching 22% in some case. Opinion of value based on Simonotti's formula may be higher than PVF of a 22%. A further analysis has been carried out increasing the holding period. The Table 6 below reports the same differences of Table 4 related to a holding period of 15 years. It shows a significant valuation variation between the methods higher than the previous one obtained for a holding period of 5 years.

In the Table 6 the column reports the estimation of d whilst the row indicates the estimation of the discount rate. It is possible to observe significative difference in term of valuation variation with low discount rate included between 0.03 and 0.05 for a d factor varying between 0.02 and 0.025. In the Table 7 below it is possible to observe the statistical data about the valuation variation between the two different models for a holding period of 15 years.

Comparing Table 7 with Table 5, the arithmetic mean of valuation variation between the opinion of value of the two methods is almost doubled. The difference in the maximum case reaches the 66% the standard deviation is almost three times than the one observed in the table 5. Consequently, the difference grows proportionally to the holding period. A further alternative is the combination of a linear and a compound variation ratio in order to create more flexibility in the valuation process. In this way it is possible to use an exponential variation rate for a certain number of years and a linear variation rate for the remaining part of the years like in the Formula 9 below:

$$V = \frac{\sum_{t=1}^{n} \frac{F_{t}}{(1+i)^{t}}}{1 - \frac{(1 \pm d)^{n} (1 \pm dt)}{(1+i)^{n}}}$$
(9)

Table 6. Valuation variation. Comparing Final Result of Simonotti's DCFA Formula with Prudential Value DCFA Formula assuming a holding period of 15 years.

Discount	Δ calculation per each value of d holding period 15 years d term					
Rate —	0.005	0.01	0.015	0.02	0.025	
0.03	0.0055857238	0.02762971	0.081992746	0.21625901	0.6683565	
0.04	0.0037092259	0.017139674	0.045817219	0.10079288	0.20785236	
0.05	0.0026794005	0.011949283	0.030447912	0.062571069	0.11623006	
0.06	0.0020341098	0.0088775096	0.022011246	0.043655449	0.077297539	
0.07	0.0015955865	0.0068639085	0.016723273	0.032457920	0.055921492	
0.08	0.001280857	0.0054539352	0.013128456	0.025115434	0.0425195207	
0.09	0.0010459832	0.00442026825	0.010547418	0.019972157	0.0334056563	
0.1	0.00086552055	0.0036365848	0.00862040582	0.0161999947	0.0268594757	
0.11	0.0007237132	0.0030270706	0.00713908177	0.0133387780	0.021969953	
0.12	0.00061028411	0.0025434687	0.00597444305	0.011112330	0.01820951	
0.13	0.00051824440	0.0021536042	0.0050423408	0.0093448617	0.0152514753	
0.14	0.000442677	0.0018352036	0.0042855514	0.0079191620	0.0128827428	
0.15	0.0003800245	0.0015723637	0.0036638145	0.006754091	0.0109584031	

Table 7 Valuation variation. Statistical Data for 15 years holdingperiod.

Arithmetic Mean	0.228676998
Maximum Value	0.6683565
Minimum Value	0.0625710695
Dev. Standard	0.223828442

In the Formula 9, the variation ratio presents an exponential form from the moment valuation to year n and a linear relation from the time n to the end of the holding period calculated as t. The PVF may be seen as a family of valuation belonging to income approach.

4. CONCLUSIONS

The paper proposed a modification to the Simonotti's formula to identify the sustainability of investments in the long term and, in particular, to calculate the DCFA Prudential Value for Real Estate lending, in the framework of the EBA Guidelines for loan origination and monitoring⁴. The variation proposes a different calculation of d emphasizing the fact that the term d has a real estate nature instead of a financial one. Therefore, as a variation of price along the time can be modelled not only using the exponential function like in the original Simonotti's DCFA formula but also in a linear way using a linear variation ratio. For this paper the method has been defined Prudential Value DCFA Formula (PVF) and allows the valuer, the investor and/or the lender to reach more prudent opinion of value. The original formula may be applied in real estate market increasing or decreasing in an exponential market; whilst the PVF may be used in specific context where these variations are weaker, or in the case in which a prudential assessment of the asset is appropriate since the time horizon under analysis is very broad and therefore the forecasts of performance in the long term are very uncertain.

This contribution shows that the previous Simonotti's model may be modified to represent better market reality and the sustainability on the value in the long term (i.e. Long-term sustainable value, LTSV)⁵. Professional operators need to be able to respond to change of external conditions, in consideration of the sustainability of the cash flows in the long term, for the entire duration of the investment (and therefore for the purpose of identifying the long-term sustainable value of the asset). The approach is functional, inter alia, for the purpose of identifying the "prudential value" as credit risk mitigant, as defined by the innovative banking regulation (Basel 3) according to the definition of loan-to-value (LTV) ratio for loan origination and monitoring. Future directions or research may be a comparison on real sample of these different relationship or introducing other possible kinds of modelling of d factor in the valuation process expanding the methodological possibilities of the formula.

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⁴ See: European Banking Autority (EBA) Guidelines on loan origination and monitoring (EBA/GL/2020/06).

⁵ About the "Long-Term Sustainable Value, LTSV" see: Bambagioni, G. (2021) "Sostenibilità del valore nel finanziamento immobiliare" (i.e. Sustainability of value in real estate financing), Maggioli Politecnica; Chapters 1, 2, 4 and Introduction.

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