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New linear models in the context of a rationalised Real Estate Valuations Science

In the context represented by the market approach, the scientific research has developed a lot of valuation models. Any typology of these models is affected by a considerable number of important problems. These models are usually considered independently from the sources of data, information and scientific basis, but this is not the correct approach. The aim of this document is to provide a synthetic analysis about the main deficiencies of the relationship between information sources and valuation models and also to propose a new linear model, conceived in the broader context of an auspiciously rationalised Real Estate Valuations Science.

Key words: *valuation, linear model, real estate*

1. Introduction

With respect to the valuation of real estates, an accurate analysis of the technical literature substantially reveals the existence of two kinds of information sources; the first one, which could be defined as *direct source*, is related to the means of providing numbers and other data types to the valuation algorithms. The second one, which could be defined as *indirect source* consists rather of all other titles from the technical literature. Both sources are affected by a considerable amount of criticisms, part of them having been just revealed and handled, whereas another part of the criticisms has not reached yet a considerable level of deep analysis. The *indirect sources* could, in their turn, be subdivided in *internal sources* of the Real Estate Valuations Science and *external sources*, which are represented by all kind of topics not strictly included in the real estate valuations. The *external sources* represent an important issue because, time after time, they develop old and new concepts, continuously updating the external basis of the Valuation Science. Depending on this fact, it should be constantly necessary to update and to improve the corpus of the scientific basis of Valuation Science, but it is really difficult to always sustain this unavoidable process. The issues, related to the Real Estate Valuations Science sources, must be investigated in the broader context of other essential aspects: 1) the identification of appropriate valuation models, 2) the application of valuation methods, which are authentically scientific (this aspect is frequently disregarded), 3) a development of the scientific research and operative applications

carried on according to appropriate criteria of understanding (i.e. Ockham's Razor) and operative instruments, correctly used (i.e. Logic), 4) combining formalism and concepts in balanced way, 5) the appropriate use of doctrinal sources, so that their errors and possible mistakes are not the presuppositions of a wrong scientific research or real estate valuations. In the described context, the non-casual encounter between the Anglo-Saxon real estate valuation doctrine and the Italian one has generated a considerable growth and development of the research about new valuation models, which could improve the capability of finding out real estate economic values. In the ambit represented by the market value approach, the linear multiple parametric models have evolved over the recent decades. In this field, the scientific research has been developed according to two distinct paths, the first one in the statistical context, while the other one is in the deterministic ambit, through several types of conceptual applications. Throughout the years, the large debates among the scientists and the numerous applications have permitted to highlight the considerable limitations of all kinds of mathematical formulae. The key reason for this fact is the discovery of appropriate linear models, which has been conducted with only a special focus on models, without taking into account the fact that the system, to be analysed, is composed by both the subsystem of providing data and information - direct sources - and the following valuation process. These two subcomponents have to be conceived as an *unicum*.

2. The indirect sources

The aspects analysed in this paper are very important, because they are strictly referring to the application of the scientific method to the real estate appraisal and because they have a lot of implications on the outcomes of studies and researches. The argument is too complex for analysing it in an appropriate manner, so that it is important to present the concepts through a simple example, derived from the past Italian traditional evaluation doctrine.

In the conclusions to his own analysis, M. Budinis[1] wrote: *"...as general indication, it is reported the result of a survey, conducted by the author during the period 1937-42 considering about 1400 buildings, in major cases located in urban ambits of a medium-size cities"*. Despite the fact that the number of buildings is certainly important, the reader can notice that M. Budinis has omitted some fundamental information: 1) the geographical localisation is not precisely defined (it is just substituted by a generic reference to the North of Italy in another part of the exposed analysis), 2) the buildings are not defined through their main characteristics, 3) it is impossible to understand which criteria was used in order to define the building population, moreover it is also impossible to understand if the population is a correct representation of the totality of the described buildings, 4) it is also unknown the subdivision between building typologies. Nobody is able

[1] (Budinis 1947, pp. 224 and following-ones).

to understand if the conclusions are good for cities like Milan, Turin, Bologna, Genoa and which are the conditions for applying the conclusions; the rest part of the analysis does not add anything of relevant. In this case nothing can be related to the *application of scientific methods*, nevertheless there is a considerable number of applicative problems.

In general, the reader, who pays attention to this fact, can note the following aspect: in the past, as well as in the recent years, an omissive behaviour is prevalent, i.e. a lot of very important information is not published with the analysis. Usually there is a huge deficit of transparency, so that it is not possible to check hypothesis and other valuation assumptions in order to verify the validity of the conclusions and the results of (pseudo)scientific researches. Considering the fundamental importance of property valuation in socio-economical ambits, this absence is really no longer reasonable.

3. The direct sources

It is not easy to propose a time partition, identifying a classic and a modern Real Estate Valuations Science, in fact in the same publication some theories maintain unaltered their validity, while others can no longer be considered as valid. Despite this fact, it was no more possible to disregard the transnational connection among other evaluation cultures and to homogenise the same concepts covered by Real Estate Valuations Science and other economic disciplines. This concept is synthetically and ably exposed by Cesare Ferrero[2], who wrote about the need of "*reconciling – giving homogeneity – the Italian tradition in valuation matters with the Anglo-Saxon and International praxis and the company-oriented valuation doctrine*". Among the others, the publication of this author is perhaps one of the most meaningful in creating a sort of disconnection between the traditional methodologies and the new valuation trends. In his publication, the sources are predominantly considered as "collecting of market data and information", so that the sources have to guarantee some requirements: 1. reliability, 2. appropriate updating concerning "market dynamics", 3. homogeneity (using several sources of data and information), 4. information specificity. Cesare Ferrero highlights that the difficulties of finding data and information can lead to incorrect operative solutions, like (i) unreliable information or not enough justified and checked, or (ii) the use of data that is not updated or is generic. In order to avoid these actions and to emphasize professionalism and strictness "*it is necessary to make explicit the data and information sources*". He also notices that "*despite the information basis can be considered a precious and expensive asset of a valuator, it is necessary to provide at least sufficient indications in order to identify the sources and to permit the identification of the quality and characters of information*". He also correctly affirms that *"in all exchange processes, rarely the beginning prices of supply and demand are the final transaction mar-*

[2] (Ferrero 1996).

ket price. The comparisons do not have to be based on potential prices, but only on effective prices of concluded transactions".

Among the most modern doctrinal papers, the *direct sources* are often recalled in the Italian Property Valuation Standard[3], which does not include a proper definition of source for evaluation purposes, but the concept is often covered, providing interesting indications. At point n. 3.8 of the chapter n. 3, on the market value, it is possible to notice that "*there may be many real estate quotations taken from different sources for the same market area, variously disaggregated and often contradictory*". At point n. 2.3 of the chapter n. 7, referring to the survey of the real estate markets, the emphasis is put on a fundamental concept, that is "*the finding, checking and rotation of the sources of data and information*" and "*the defining of data quality control procedures*". At point 3.1 of chapter n. 7 it is analysed in deep the reliability of the direct sources, noticing that "*it is well-known that in Italy the veracity of data is a complex problem, which concerns a widespread behaviour among declarants, who tend to indicate in deeds of sale a lower price than that the actually negotiated*". At point 2.2 of the chapter n. 7 it is brought to a specific focus on the real estate market survey requirements, observing that "*the collecting of property market data must have the following requisites:*

- *the veracity of the property data, in the presence of the widespread practice of concealing the prices actually negotiated;*
- *the completeness of all the elements making up the property data; the property data is made up of an economic part regarding the price actually paid, and a technical part regarding the positional, structural, typological, and technological characteristics of the property;*
- *the accuracy with respect to the diligence, the competence and the precision of the data collection".*

At point 3.6 of the same 7th chapter it is expressed the opinion that "*in property data collection operations, it is necessary to have a standard for evaluating the quality of the property data, as concerns the requisites and the veracity of the property data and the procedures for collecting, filing and consulting these data. The proposal of a property data standard derives from the lack of any primitive property standard and from the importance of the evolution of the Italian real estate market towards integration with the securities market*".

Among the publications, which deal with the issue of the interaction among the Italian tradition, the Anglo-Saxon and International praxis and the company-oriented valuation doctrine, there is an interesting book, signed by Alberto M. Lunghini[4]. He associates the quality of the valuation to some variables: *1.*" *the competence of the valuator, 2. professional and moral integrity of the valuator, 3. availability of data, 4. ways of surveying, selecting, checking and interpreting the available data*". It is clear that these aspects are not completely independent variables; in fact the surveying, checking and interpreting processes are strictly dependent on the valua-

[3] (Tecnoborsa - third edition).

[4] (Lunghini 1993, pp. 67 and following-ones).

tor's competence and on his moral and professional integrity. The aforementioned causes are four essential elements for obtaining an acceptable result; they must always be considered together in each scientific and practical application. From the chapter that presents the history of the modern evaluation theories, it is important to notice that "*generally it is not available a sufficient number of meaningful data for using advanced statistical models*", but Alberto M. Lunghini argues that despite this fact it is possible to consider the valuation of real estate not only as a simple procedure, but as an application of foreseen models. Another important concept is the remark that "*last year of the twentieth century imposes the essential financial structuration of the valuation science*". These two last aspects together have heavily conditioned, in the recent years, the surveying of data and information, which is an important part of the valuation science sources.

4. Valuation models in the Real Estate Valuations Science

According to the "ratio" of the European Directive Inspire[5], the cadastral parcel, which can usually be associated with an economic value or rent, is considered as a locator. Numerous European countries use the cadastral information for taxation and valuation purposes[6]. These facts are only two related examples of the broader concept that states that it is possible to conceive the economic value as a function of same variables[7]. According to the previous observation and other circumstances, it is possible to assume that the economic value can be considered as a continuous function of some characteristics, but the main dependence comes from the dimension of the single real estate, usually expressed in square metres or other measure units. Focussing on the Market Value, one usually calculates this as

$$V = V_p(y_1; y_2; ...; y_n) \cdot A \tag{1}$$

where y_1 up to y_n are measurable characteristics, related to the real estate, and A is a parameter connected to its dimension (like area, volume, etc.), which can be determined through specific codified rules. For instance, in Italy the most common rules are included in the annex C of the Decree of the President of the Republic n. 138/1998. According to the *Ockham's razor*[8] the valuation model must

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[5] Directive 2007/2/EC of the European Parliament and of the Council of 14 March 2007, establishing an Infrastructure for Spatial Information in the European Community.

[6] *The cadastral parcel in NSDI's and in INSPIRE* – http://www.vector1media.com/news/top-stories/53-corporate-news/4341-report-the-cadastral-parcel-in-nsdis-and-in-inspire-.html

[7] Firstly a brief localisation, usually expressed through the inclusion into a homogeneous area, where the real estate entities have approximately the same characteristics.

[8] There are some versions of the Ockham's razor; usually it is reminded as a sort of principle. Some versions were not written directly by William of Ockham, but they must be considered

be the simplest one and the number of the variables (characteristics) must be the lowest one. These two consequences of the *Ockham's razor* consents to define the choice of linear models, where the unit value is a function of some technical or economic parameters (independent variables).

In a simplified approach, if each addend $(a_i x_i)$ is conceived as a contribute to the composition of the unit value and x_i is also conceived as a parameter for the amplification of a_i, which consents to a valuator to determine and to govern the variability of each comparative character (independent variable), then it is possible to conceive a_i as a specific contribute (percentage) to V_p, appropriately managed through the auxiliary parameter x_i. As a consequence, the unit parametric value can be expressed through the linear combination, having put $x_o = 1$:

$$V_p = a_0 x_0 + a_1 x_1 + ... + a_{n-1} x_{n-1} + a_n x_n = \sum_0^n a_i x_i \tag{2}$$

The use of linear models is usually linked to the direct market comparison approach[9], but, in general, in other valuation cases there are effectively not acceptable reasons for using more complicated models without appropriate scientific data and information. Consequently, the strong difficulties of modelling the very complex technical and economical situation, related to the valuation of a real estate, usually leads to the easiest typology of model: the linear model. This circumstance is one of the indicators that point out the enormous difficulties specific to the Real Estate Valuations Science and its extreme characteristics, not presented in other common sciences.

It is now important to make some remarks. First, plenty other mathematical functions can be directed to the previous linear expression; the linear models are usually considered as real-valued continuous functions, where each variable has its arguments included in its domain. On its turn, each variable can be considered as a function of other variables. For instance:

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as posterior improvements of the original concepts. In the modern approaches to sciences, the same original principle is frequently proposed three times, so that it is possible to avoid difficulties in the analysis of theories, by having an immediate comparison with the most appropriate version of the Ockham's razor. Probably the most common version is the following one: 1. *entities must not be multiplied beyond necessity* (entia non sunt multiplicanda praeter necessitatem), 2. *Pluralities must not be considered if it is not needed* (Pluralitas non est ponenda sine necessitate), 3. *It is pointless to do with more what can be done with less* (Frustra fit per plura quod fieri potest per pauciora). Other common statements of the Ockham's razor are: a. The simplest explanation is usually the best, b. The simplest explanation that covers all the facts is usually the best, c. Concepts are not to be multiplied beyond necessity—the corollary of which is: nor are they to be integrated in disregard of necessity.

[9] In this meaning the market comparison approach is intended, according to a part of the classic Italian literature, as a criterion, applied through a certain number of methods. One of these is the statistical approach (linear regression) and another one is the *data matrix approach*, a method that in some Anglo-Saxon literature has the same name of the (Italian) criterion.

$$V_p = b_0 \cdot b_1^{y_1}. \tag{3}$$

In this case, through some mathematical passages[10], the original valuation model is directed to the linear function

$$v_p = a_0 + a_1 y_1, \tag{4}$$

where $v_p = \log V_p$, $a_0 = \log b_0$, $a_1 = \log b_1$. Particularly this mathematical possibility allows to treat some kinds of models through the linear regression.

Considering a linear valuation model, it is also possible to observe that linear models can have the variables formulated as function of other variables, for example the unit parametric value can be considered as $V_p = V_p(y_1; y_2; ...; y_n)$, but $y_i = y_i(x_1; x_2; ...; x_m)$. It is the case of "function composition"; if all the functions are linear, then the composed function is linear too. According to the Ockham's razor, variables must not be multiplied, if this operation is not needed, so that the introduction of m new variables can be admitted if $m < n$, but the reader has to remind that the more simple explanations are *generally* better than the more complex ones, but only if other *aspects* (factors, conditions, etc.) *are equal.* As a consequence, a higher number of new variables (comparative characters) can be accepted only if this fact produces general better conditions, like higher precision degree, better management of the argument variability, etc. Effectively it is possible to find out in the real estate valuation literature several forms of linear models, sometimes some of them is derived from others through the substitution of some variables (comparative characters), using a composition of functions. It is quite frequent that the exposed theory does not comprise explained methods for the appropriate management of the newly introduced variables. The models that are obtained in this way cannot be accepted, because they do not really allow reaching a better precision degree and they do not improve anything; on the contrary, they only introduce useless complications; sometimes reasonable and appreciable models are simple impressions. Here is an example.

Sometimes in the real estate valuation literature and in the valuation practise the rate of the capitalisation formula is calculated through an astonishing linear model:

$$r = \sum_1^n r_i \cdot p_i \tag{5}$$

where, for instance, r_1 indicates that the rate is calculated with a different algorithm from r_2 and p_i is a percentage, associated to the condition $\sum_i p_i = 1$.

[10] $\log V_p = \log(b_0 \cdot b_1^y) = \log b_0 + \log b_1^y = \log b_0 + y \cdot \log b_1 = a_0 + a_1 y_1$.

In the aforementioned linear model there is a fundamental conceptual error, namely that the entities (algorithms and independent variables) are multiplied beyond necessity; the use of the parameter p_i, without rules for managing p_i, seems a trick in order to obtain a just predefined rate, which is intended, rather than a scientific approach.

Usually the multi-parametric calculation procedure is strictly related to the linear regression method, by following the steps listed here below:

- definition of the comparative characteristics, which have an influence on the comparison process and which can be - directly or indirectly - quantified;
- to set up a specific number of multiparametric linear functions in order to define the unit parametric value;
- considering similar real estates, the definition of a sufficiently numerous statistical population;
- the linear regression techniques are applied in order to define the dominium of the independent variables and the coefficients of the linear combination.

The aforementioned procedure could seem very easy, but it is not at all, because the use of statistical methods brings a lot of undoubtedly advantages, but also a considerable number of disadvantages.

First, one can not exclude the presence of some outliers, for instance real estate units affected by anomalies, which do not consent to the same real estate units to be represented by the multiparmetric linear model and, if they are considered, outliers can deteriorate the mathematical formula. In general the outliers can be detected through statistical tests, but their withdraw from the real estate unit population creates more and more complex procedures.

The statistical procedure is also extremely redundant, if the goal is the determination of the unit parametric value of only one or few real estate units. As a consequence, their use is more useful in the "Computer-Assisted Mass Appraisal systems" (C.A.M.A.). Another disadvantage is represented by the fact that the calculations are very complicated, so that it is indispensable to use specific statistical IT applications, which are mostly very expensive[11].

Linear models are approached not only in the linear regression; there are other methods of managing the linear models, all these last ones can be included in only one typology, usually named *data matrix approach*, so that other methods designed as *market comparison approach, base property approach, market data adjustment grid method, market comparison method, paired sales adjustment method* can be considered variants of the same approach. The fundamental aspect in these techniques is the following one: one analyses couples of sales in order to reveal the difference in value that is due to the presence or the absence of attributes, precisely between the real estate subject of valuation and the other comparable real estate units of the selected population (for the comparison). The technique

[11] For further details about criticisms in linear regression approach, it is recommended to the reader the analysis signed by Marco Simonotti. (Simonotti 1997).

examines the adjustment of values either on the basis of a currency amount or as a percentage amount. In this approach the linear combinations[12] are ordered in a matrix, which is developed for a number of properties and features (which represent the determinants of value) are identified for each single property. The data compared analysis should lead to the quantification of the differences between the real estate subject and compared ones, which consent to calculate the subject market value through some easy mathematical passages. There are a number of theoretical and practical problems associated with this kind of analysis. First, it is almost impossible to conduct the comparables to the subject in order to obtain two properties that are identical in all characteristics[13]. It is also difficult to identify the influence on the value of some of the comparative characteristics; in the real estate valuation literature the reader can notice that in many respects the combination of two or more factors can have a synergistic effect. Usually the population used for comparative process is derived from an unique homogeneous territorial area. A fundamental implicit supposition (evaluation hypothesis), common to all the techniques strictly related to the data matrix approach, is that all the comparative characteristics have the same percentage influence on the value of each considered comparable real estate. This fact should be demonstrated or appropriately justified, because in general the contrary is true[14]. The data matrix approach has a considerable number of conceptual deficiencies, which are more or less apparent, depending on the methodological version; an interesting synthetical analysis has been conducted by G.B. Cantisani[15]. In his paper he has summarised some of the most criticisms that have been analysed in the technical literature: 1. problems regarding the definition of marginal prices, 2. sometimes the market comparison approach is used with just too few comparable real estate units (only 3 or 4 comparables), 3. if the variability of the comparable characters is too large, the results are often affected by a degree of imprecision that is not acceptable, etc..

Both approaches have some common criticisms. Considering that the value is always referred to a well specified moment, in some territorial contexts the number and the typology of comparative characters can vary[16] over the time. The acoustic pollution is given as example, for being more and more a remarked social aspect. If in a specific homogeneous market area it is built a new motorway, it can produce a new unsustainable level of acoustic pollution, which probably generates effects on the values of the nearest apartments, but does not affect the remote properties. It this case, for new valuations of apartments it will be necessary to redefine the valuation models and to add new comparables and it is not always so easy.

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[12] Each one is a linear equation that, in origin, expresses the value of each real estate.

[13] That is, the identified characters of the comparison between the comparables and the subject and, also, the other characters not directly involved in the comparison analysis.

[14] (Orefice 1995, vol. II, pp. 45-46).

[15] (Cantisani 2008).

[16] Apart from the fact that the mutual importance of the chosen comparative characteristics can also vary.

Another criticism derives from the percentage influence of the parametric unit value of each characteristic; as indicated, each independent variable is an addend of the linear model and each of them can have a low or a high percentage influence on the value. It is quite usual to prefer those characters that have the highest level of influence, but this aspect depends on the valuator's subjective choice. A higher number of characters (independent variables) usually generates a higher level of precision, but also a higher number of comparables. Sometimes the choice is not easy: the number of comparables could be insufficient for the precision that is needed. On the other hand a low (available) number of comparables could conduct to an insufficient level of precision.

5. Two examples for thinking over

In order to understand better some concepts about the relation between the capacity of a real estate population of being representative and its influence on the conclusions and results, here are two examples.

The reader takes as hypothesis the fact that a homogeneous market area has been surveyed; more precisely, a real estate segment has been surveyed and this segment is composed by apartments that have the same comparative characteristics. As a hypothesis (for example as a consequence of a very detailed administrative and scientific activity), all the parametric unit prices of the transactions of a specific period are considered known. The reader considers to assign to two groups of researchers the task of surveying the considered market area[17], by casually extracting ten transactions for each group. All the transactions are included in the table n. 1, where it is also drawn the frequency diagram. First of all, it is interesting to observe that the statistical variable is bimodal and appreciably different from casual variables as Gauss, Student, Poisson and Binomial distributions. This example would need of a more developed analysis, which can not be approached in this short document; nevertheless it is possible to run a reasoning process, only by addressing the statistical mean[18]. For each casual extraction (of ten transactions) from the statistical variable it is possible to calculate its mean, which can vary from its own minimum and maximum values. In the proposed example, the mean varies from 3,350 monetary units (m.u.) up to 3,720 m.u., which has for result the following percentage differential

$$\frac{3{,}720 - 3{,}350}{3{,}720} = 0.0979 = 9.79\%$$

that is about 10%. In the most knowledgeable traditional doctrinal presentations, ten is usually considered the lowest number of real estate units for calculating in

[17] Providing human, financial and instrumental resources.

[18] The sciences usually designate this sort of simplified explanation as *intuitive justification*.

an appropriate manner the mean, but, as showed, the variability of the mean noticeably influences the market values[19]. It is now necessary to make some remarks. First of all, the mean can be interpreted as a positional index of the entire population, while variance measures the statistical dispersion; so that the omission of the variance analysis is justified by the need of preserving the generality of the reasoning (because there are serious criticisms in the adoption of statistical models for valuations and also because the mean itself has a great influence on the conclusions of the exposed reasoning).

Now it is considered as being a consecutive period, so that the two periods have the same temporal extension, i.e. six months or one year. Also in this second period all the unit market prices of the homogeneous market area are considered known; the research teams also survey the second period, by using the same methods as in the first case. Comparing the two periods, it is observed a substantial stability of the real estate transaction number. In the second period the mean varies between the minimum of 3.320 m.u. and the maximum of 3.730 m.u., so that the percentage differential is

$$\frac{3,730 - 3,320}{3,730} = 0.1090 = 10.90\%$$

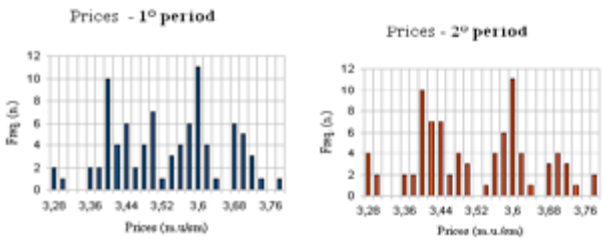
Assuming that the mean can be considered a representative index of the market prices in the two analysed periods, the reader can observe that, depending on the choice of the statistical population, one can obtain a price contraction of 1% up to a more intense increase of the 11%. The conclusion of this reasoning is also based on the hypothesis that any other type of market influence is negligible or, at least, very small. The example clearly shows the fundamental importance of the real estate market survey, which shall conduct to a real representation of the market by the use of reliable data and information. It is not worthless reminding that the real estate market survey is indispensable in order to apply correctly the valuation models. The application of scientific methods to valuation models does not guarantee by itself the delivering of corresponding conclusions with the reality of the market. Consequently it is possible to better understand that the subjective component, unavoidable included in every scientific application, can sensibly influence the entire process of the technical and economical modeling. It is clear that model results and market surveying depend on the researchers' and valuators' technical and scientific competence, but also on the fundamental importance of ethical and deontological irreproachable behaviors, free from personal and general interests. The simple authority to address the real estate market survey activities in a specific market ambit can have an influence on the trend of the real estate market. The researchers' and valuators' correct ethical and deontological behavior is usually an implicit and unexpressed hypothesis of any valuation process; but, as

[19] It is well-known that mean is a positional parameter, that defines the position of the related population.

Table 1. Trend analysis of real estate transactions in two consecutive periods of an homogeneous marked area.

Transaction Prices (monetary units x 1,000)	Frequency – 1st period (n)	Frequency – 2nd period (n)
3.28	2	4
3.30	1	2
3.36	2	2
3.38	2	2
3.40	10	10
3.42	4	7
3.44	6	7
3.46	2	2
3.48	4	4
3.50	7	3
3.52	1	0
3.54	3	1
3.56	4	4
3.58	6	6
3.60	11	11
3.62	4	4
3.64	1	1
3.68	6	3
3.70	5	4
3.72	3	3
3.74	1	1
3.78	1	2
Total number	86	83

Frequency diagrams



much as possible, it is always opportune that, where it is possible, the market data and information users consider (i) the reliability of direct and indirect sources, (ii) the valuation process and (iii) the interests of the persons and of the structures involved in this process.

The concepts related to the second example can be easily exposed through the main part of a personal dialogue between the Author and another international expert (not Italian), that took part during a professional meeting. The object of the discussion was the indistinct and massive use of the *market comparison approach* in mass valuation systems for taxing purposes. The international expert substantially sustained that market prices, indicated in transaction deeds, can generally be considered an adequate source for the massive use of the data matrix approach. The exhausting discussion ended up in an unexpected way after the question: "*Are you able to demonstrate scientifically or to justify appropriately that never in your Country*[20]*, in parallel with the signing of the transaction deed, there is no other kind of money transfer*?" He unexpected and sincerely replied: "*No, I can't*". Independently by the possibility of using the market comparison approach to mass valuation systems[21], there is a conceptual and practical problem with the direct sources; it is always necessary to formulate a rigorous social and valuation hypothesis: *in each deed, related to a transaction of properties, the indicated amount of money is the concrete total of the transferred amount*. Perhaps in some Countries, or some parts of them, this hypothesis could be validated as being true, but at the worldwide level there are still some regions where the transactions are still based on verbal agreements. Furthermore this hypothesis has a social impact and implications: in some countries the application of the market comparison approach, supported by the rates indicated in the deeds, could provide very important deficits in fiscal withdrawal. It is always very important having a clear and complete vision of all the implicit and explicit hypothesis of a valuation process.

6. A new kind of linear valuation model

Taking into account the fact that the linear regression and the data matrix approaches are subjected of numerous important criticisms, a researcher could ask himself if it is possible to find other algorithms for solving the valuation issues without using them. It is not conceivable to use non-linear models in the Real Estate Valuations Science, because of all its peculiarities, especially the number and the typology of the comparative characteristics; these are not always known in advance. For instance, in an apartment located on a hill near the seaside, a panoramic view could have the main influence on the value; meanwhile the value of the same apartment, located in a mountain village, could be influenced by the

[20] A well-known situation.

[21] There is a considerable number of reasons for not using it, what exceeds the goals of this document and can not be appropriately exposed.

distance to the ski-lifts, instead of the panoramic view. So, in general, neither the number, nor the typologies of comparative characters are predetermined; this situation is very different from the case, for example, of Thermodynamics, where pressure, volume (specific to the mass) and temperature univocally define the "thermodynamic state" and these variables are reciprocally connected in the "state equation[22].

The reasoning begins on the evidence that the evaluation of a specific real estate is done on the basis of an appropriate number of population of similar real estates. Some other valuation hypotheses are *a priori* considered as being true, for example the existence of a real estate market or that the entrance on the market of the valued real estate does not alter the same real estate market.

Each property of the population has its useful comparative characteristics, univocally defined; then, each comparative character must be appropriately quantified in the ambit of a proper measure scale. This measure can be more or less objective and easily quantifiable, according to the aforementioned scale or it could be obtained through an evaluation. For instance the *maintenance* of the real estate could be related to some steps, each one can describe a specific state of this comparative attribute, for instance "well maintained", etc. This process generates a matrix, its columns describing the association of the comparative characteristics to the same real estate and its rows revealing the type of variability of a specific character in all real estate units of the population. Each of these comparative attributes has, on the unit parametric value (incognito), its own percentage of influence, which varies between its minimum and maximum. At this point of the explanation it is important to make other considerations. This type of linear model has been conceived by the Author as a part of an innovative theory, which proposes new conceptual basis about what in the Italian traditional Evaluation Science is commonly called "teoria dell'ordinarietà"[23]. This way of conceiving the ordinary includes the concept of metric scale of evaluation, which can be used for each comparative character (by the subdivision between the minimum-maximum interval in an appropriate number of steps). For each comparative attribute, the juxtaposition of the same characteristic (of the subject) in a specific step of the evaluation scale[24] permits to define the contribution of each addend in the context of the linear combination.

The transposition of the aforementioned concepts in a linear model must be completed by other assumptions:

- disproportionate complexity of the actual multiparametric evaluation methods[25], considered in relationship with the usual evaluation scopes;

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[22] Despite this fact, there is a sort of similitude between the thermodynamic and comparative characteristics and also between the state equation and the evaluative equation.

[23] The explanation of the whole evaluation theory obviously exceeds the goals of this short document; however this typology of linear model is conceptually a very flexible instrument, which can be easily adapted to other structured evaluation theories.

[24] Each comparative characteristic has its own evaluation scale.

[25] *Linear regression* and *data matrix approach*.

- questionable reliability of results obtained to linear regression and data matrix approaches;
- no real scientific justification by the use of non-linear models;
- adoption of the more recent doctrinal typological subdivision of the comparative characters[26], as minimal partition for valuations;
- awareness that the influence of each comparative character generally varies between a minimum and a maximum values in the same homogeneous market area;
- converging to a linear model, which could be considered a derivation and synthesis of the principal Italian traditional valuation procedures;
- existence and generalised use of codified measure system[27] of area, volume, etc.

The precedent assumptions lead to the conclusion that the parametric unit value can be divided in its partitions and that a comparative analysis can be conducted on each one; every partition can be composed by one or more characters, if the aggregation can be correctly evaluated in a unitary way. Furthermore, thanks to this approach, not only the main characters are compared, but also other important (even if not fundamental) attributes.

According to the precedent explanations and to the summarised analysis, the evaluation procedure can be subdivided into two parts: 1. the calculation of the area, volume, etc. under the terms of codified rules[28]; 2) the determination of the parametric unit value.

The second part of the procedure contains the steps listed below.

- Recognition of the comparative characters that have an influence on the comparative process between the comparable real estates and the evaluated property.
- Aggregation of comparative characteristics in homogeneous classes and removal of characters without influence[29].
- On the basis of the comparables, for each aggregation of attributes the valuator should determinate the minimum and maximum influence on the parametric unit value[30].
- Comparing the real estate units of the population with the real estate subject of evaluation, the valuator is able to modify objectively the value quote, aggrega-

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[26] In market comparison approach it is usually adopted the subdivision of characters in: 1. *localisation* – in order to manage the relationship with the territorial context; 2. *position* – in order to differentiate the property units in the contest of the buildings; 3. *typological* – in order to manage attributes specific to the property units (maintenance, etc.); 4. *productive* – in order to manage residual characteristics linked to the possibility of economic returns and so on.

[27] As just mentioned, in Italy the most used rules are included in the Decree of the Republic President n. 138/1998.

[28] For length reasons, this aspect of the procedure, very easy, can not be exposed in these notes.

[29] The aggregation of the characteristics is not necessary univocal, but usually there is more than one solution.

[30] In the evaluation theory, approximately summarised in this article, this fundamental aspect should be assigned to a specific structure for monitoring and analysing the real estate markets, using transparent and appropriate methods.

tion by aggregation. The value quote can be assigned to each classification group; adding all the value quotes it is obtained the parametric unit value.

In practice, the parametric unit value is obviously obtained as linear combination of parametric unit values assigned to the single characters (or aggregation of characteristics) and the related parameters, which adjust the addends of the linear combination to the valuation case. In general, each parametric unit value (related to its character or aggregation of characteristics) can be obtained as percentage of the whole parametric unit value. As aforementioned, in the real estate population, each single parametric unit value varies between its own minimum and its maximum value. In order to manage appropriately this part of the process it is necessary to identify a "graduated scale of intensity", which essentially respects two distinct requirements:

- the scale must be subdivided in a sufficient number of attribution classes, in order to obtain a meaningful classification;
- the number of classes must permit that the valuator's judgement consents an unambiguous juxtaposition of the evaluated real estate in an appropriate step of the "graduated scale".

The second requirement must guarantee the minimum level of subjectivity in the valuation judgement, so that the same estimated real estate could be referred to a determined aggregation of characters, rather than another, independently by the individual judgement of two or more valuators.

The aforementioned requirements are summarised in the choice of a limited number of evaluation classes in the "graduated scale of intensity"; for instance it could be suggested to define six classes, which coefficients can vary from zero up to five. The interval of values is not limited by zero and five, the use of coefficients like six or seven permits to calculate parametric unit values also for very particular real estates. For each class of attribution[31], if the index number is zero, then its input to the parametric unit value is the lowest and if the index number is five, then the mentioned input is the highest (for common real estate units).

In order to introduce the proposed new linear model, it is also necessary to define some parameters:

$V_{u\max} = V_1$ is the maximum ordinary parametric unit value,

$V_{u\min} = -V_2$ is referred to the minimum ordinary parametric unit value,

j = number of classes = 1,2, 3, …, m,

i = 1,2,

c_{ij} = percentage incidence of the j class on parametric unit value (minimum or maximum),

p_j = *merit coefficient* of the j-class[32],

Vu = parametric unit value of the evaluated real estate (incognito).

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[31] For instance: 1. localisation, 2. position, 3. typological, 4. productive.

[32] All the *merit coefficients* are a direct consequence of the considered real estate evaluation activity.

Using the above definitions, it is possible to build the below listed matrixes:

$$V = \left[\begin{array}{cc} V_{u\max} & - V_{u\min} \end{array} \right] = \left[\begin{array}{cc} V_1 & V_2 \end{array} \right]$$, called "*value vector*";

$$C = \left[\begin{array}{cccccc} c_{11} & c_{12} & \ldots & \ldots & c_{1(m-1)} & c_{1m} \\ c_{21} & c_{22} & \ldots\ldots & \ldots & c_{2(m-1)} & c_{2m} \end{array} \right]$$

"*coefficient matrix*" of the (statistical) population (or market homogeneous area)[33],

$$P = \left[\begin{array}{c} p_1 \\ p_2 \\ \ldots \\ \ldots \\ p_{m-1} \\ p_m \end{array} \right]$$ "*merit coefficient vector*".

Under the terms of the precedent definitions, ΔV_u is determined through the matrix linear model

$$\Delta V_u = \frac{1}{n} \cdot V \cdot C \cdot P \tag{6}$$

where "n" is the number of steps of the "graduated scale of intensity".

At last, the parametric unit value is calculated as[34]:

$$V_u = V_{u\min} + \Delta V_u . \tag{7}$$

The total value of the considered real estate is then determined by multiplying the parametric unit value for the dimension of the single real estate, usually expressed in square metres or other measurement units.

In their simplest (usually suggested) application, the comparative attributes can be aggregated in four classes, i.e. according to the subdivision in localization, positional, typological and productive characters.

A meaningful moment in the valuation procedure is the definition of the "*coefficient matrix*"; as just mentioned it would be better that the coefficients are deter-

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[33] For instance, see Orefice 1995, vol. II, p. 44-46.

[34] This valuation model and the related concepts are included in an opera protected by the Italian law (S.I.A.E. – OLAF Sect.). If the reader is interested in, he is always authorised to use the model, respecting two conditions: 1. using it in non-commercial applications, 2. clearly citing this document in explicit way.

mined *a priori* for the whole homogeneous market area. In general, these coefficients are not constant[35], in some cases the homologous coefficients[36] do not differ so much, so that they can be considered constants[37], as always wrongly conjectured in *data matrix approach*.

For a better understanding of this linear model, it could be useful to put in place a very simple application, through the worksheet here below (table 2); its comprehension is very easy and the reader can approach it without any superfluous comment.

Table 2

Homogeneous market area		Description of the homogeneous market area					
Classification of attributes		localisation	position	typological	productivity	check (%)	
Max. value (m.u.)	2,000	30%	20%	20%	30%	100%	
Min. value (m.u.)	1,000	30%	20%	20%	30%	100%	
Merit coefficients (0-5)		5	3	4	3	1,760	=Vu

The main advantages of this kind of linear models are manifold. First, the deficiencies of linear regression and data matrix approaches are overtaken. Secondly, the matrix structure consents to treat mathematical passages using the matrix formula, regardless from the number of attributes. The reader has also to consider that this linear model is very "flexible", it is easy to graduate the complexity of the model to the practical goals. It is also possible to evaluate easily no-ordinary real-estates[38]. The flexibility is also demonstrated by the possibility of using the model both for structured mass valuation systems and for single valuations; for instance, developing a C.A.M.A., it is not compulsory to force no-common properties in staying into minimum-maximum interval. Last, but not least, this type of model is conceptually very easy[39], also in its operative applications, where the few resources, that it uses, make it very interesting, affordable and cheap in use.

Conclusions

During these last decades, in the valuation of real estate units it has been paid a special and excessive attention to the valuation models. The complexity of the technical and economic reality, which has to be transposed and summarised in

[35] (Orefice 1995, vol. II, p. 45-46).
[36] Of the same comparative characteristic aggregation.
[37] This fact is a limit of data matrix approach, this type of linear model overtakes the limitation, considering the variability of coefficients.
[38] The minimum and maximum values are not impassable limits.
[39] The Author considers the Ockham's razor as a fundamental instrument for the scientific research.

appropriate models, has reasonably conducted to an extended analysis of linear models. Going beyond the traditional approaches, especially in the market comparison approach the researchers have developed two kinds of models: the *multiple linear regression* and the *data matrix approaches*. Both approaches are subject to a lot of conceptual and practical criticisms. Considering the intention of going beyond these problems, the Author has developed a new formulation of linear valuation model for the direct comparison approach, but it has to be considered that, in general, the excessive, almost scientifically pathologic and not sufficiently justified, interest for linear models has its probable origin in a wrong assumption: better (linear) valuation models produce more precision and awareness in determining economic values of real estates. This is not completely false, but the correct perception of the reality needs to consider it from other perspectives.

First of all, valuation models are built up on two fundamental bases. The models themselves are a consequence of the results of researches and assumptions derived from evaluation, technical and economic theories. Often, these theories are based on questionable hypothesis, sometimes theories on the same argument are even in reciprocal contradiction. On the other hand, for giving appropriate and realistic outputs, valuation models need correct inputs, namely data and information. These are a real very important issue, which is not totally solved yet. A simple analysis of the data and information about the homogeneous areas of the same real estate market often reveals lacks in transparency, insufficient level of consistency of information and data, etc. Often the way of determining the statistical populations is not clear or is completely omitted, usually it clearly appears that data and information are derived from statistical analysis, but it is not usually clear what kind of distribution model is used, which are the hypothesis and so on. The data and information are given as numbers without any kind of justification, as a revealed truth. Different ways of analysing the same real estate market area can conduct to very different results (of the same valued subject), even if they use the most appropriate valuation models. In some cases the valuation theories put their basis on too old approaches, like the abuse of the "normal distribution", which aprioristic extensive use is no more accepted since some decades without real confirmation[40].

The unavoidable limits of direct and indirect sources and models must be considered together with the "human factors". Among the others, Giorgio Israel[41] is able to describe the connection between scientific models and the scientists' ethic and skills in an expressive and simple way: "*Nowadays, often, models are conceived for everything and the correct principle of adapting to the intrinsic logic of the analysed object often degener-*

[40] (Orefice 1984, p. 154). "*Above all, it is necessary to have a sufficiently numerous population of data and information related to comparable real estates, also for checking the typology and the shape of the frequency distribution of comparable technical and economical characteristics. The real frequency distribution of the found out data does not always permits of drawing an unimodal and symmetric mathematical graphic, as the Gauss distribution: in fact, it is quite usual that the real distribution is rather different from the normal distribution*".

[41] (Israel 1986, p. 113).

ates in the most absolute arbitrariness". It is reached a sort of self-referential modelling, which justifies and supports itself; Giorgio Israel continues*: But the difference between adapting to the intrinsic logic and the admittance that a model justifies itself is very wide! Despite this fact there are persons who have the courage to sustain it. …. omitted … In short, each model, without a linkage with the reality, is itself justified. So, we are free of inventing what we prefer …. It is superfluous to affirm … that this kind of models is preferred by people who want to obtain results without breaking the brain and, above all, without running the risk of breaking the brain against the reality*". The well-known scientist concludes: "*On the other hand, every back step is a non-sense. Coming back to the dream of a unification, based on Physics principles and methods, is only a useless nostalgia. We have to accept the variety of methods and approaches of today Science, included all generated risks and dissatisfactions*". Since few decades, shared modellistic solutions, which are strictly obtained from the scientific method, do not exist; on the contrary there is an offhand application of pseudo-scientific methodologies, often using self-referencing models. In this contest it is astonishing to observe that, between infinite kinds of valuation linear models, the scientific research has concentrated its attention only on the aforementioned linear regression approach and the so called "market comparison approach", nevertheless they are subject to a well-known considerable number of criticisms. Considering these limits, the Author has developed an alternative new type of valuation linear model; this third way of conceiving linear models brings alone a lot of advantages like flexibility, lightness, easiness, affordability in applying it, etc. Taking into account these circumstances, it is proposed a new structure of the multi-parametric linear model, conceived as a synthesis of some aspects: 1) similarity and synthesis of the most important traditional valuation procedures, 2) evolution of the actual Italian cadastral mass valuation system, 3) easy formulae and applications, etc. This kind of model is a sort of compendium of both Anglo-Saxon and Italian real estate valuation traditions, unified in a very flexible operative instrument, which can be used for mass valuation systems or for single real estate valuations.

All mathematical models of scientific disciplines depend on the existence of an appropriate data and information basis, so that also the best scientific models can provide correct or incorrect results depending on the existence of adequate structures for the analysis of real estate markets, which is still a not solved issue. Any valuation model is not able by itself to provide correct results without using appropriate data and information, derived from suitable market surveying procedures and activities; effectively, this two related aspect must be analysed as an *unicum*. Generally there is no transparency in surveying methods; the hypothesis are not clear and sometimes no more dimly recognised as true (i.e. the Gauss distribution applied everywhere); occasionally there are contradictions between hypothesis and conclusions and also between the results of several surveys on the same homogeneous market area. It is no more avoidable to approach correctly the relationship between valuation models and market surveying and, in short terms, to converge to a new idea of *land and real estate market information management system*, integrated with appropriate valuation models: the generation 2.0.

The proposed new model has been conceived for supporting this new kind of market information system in a simple way, however its use is appropriate

for old-structured information basis too, but, auspiciously, its use will provide a particular utility, using new appropriate information sources. Anyway, the final, logical and unavoidable consequence is the strict necessity of rethinking all the Real Estate Valuations Science, that can not be avoided any longer and that can be done by the analysis of the whole system, composed by the valuation sources (real estate information systems, technical literature, etc.) and the models.

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