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Key words: compound values; valuation error; building area

# The valuation error in the compound values

In appraising the "valore di trasformazione" the valuation error is composed by the error on market value and the error on construction cost. In appraising the "valore complementare" the valuation error is composed by the error on market value of complex real property and on market value of the residual part. The final error is a function of the partial errors and it can be studied using estimative and market ratios. The application of the compounds values to real estate appraisal misleads unacceptable errors if carried out with the *expertise*.

## 1. Introduction

If a real property presents one or more potential transformations, or it has an economic link to other real properties or their parts, or it performs instrumental tasks in the production, or if it is in an intermediate stage of a construction cycle, the market value, according to the valuation standards (*IVS - Concepts Fundamental to Generally Accepted Valuation Principles 4.10 - Introduction 3.0*), is estimated with compound valuation's criteria such as the "valore di trasformazione" (*VT* that is the difference between the market value of the transformed property and the construction costs) and the "valore complementare" (*VC* that is the difference between the market value of the total property and that of the residual part). The reliability of these valuation depends: on the precision of the market value of *VT*; on the precision of market values of the part and the whole property, in the case of *VC*.

International valuation standards put the emphasis on precautions to the use of such criteria in real estate appraisal.

In the *VT* and the *VC*'s appraising, the valuation error has a dual nature linked, in the first criterion, to the market value and the construction cost and, in the second criterion, to the market value of the real property and of the residual part. The total error is a function of the partial errors. The study examines the valuation errors in the methodological conditions posed by the *VT* and the *VC*.

The discussion on the valuation error can find an analytical formulation in the market value of the building areas appraisal. In this case the valuation error con-

cerns the expected value of the property, the estimated construction and promotion costs, and the expected profit's rates. Each of these valuations leads to a partial error, which can be computed by comparing the total error: the estimated value or values with the market price or prices, the estimated costs with final costs, the expected profit with the profit achieved.

Valuation error of compound values is obtained by a functional relationships which include parameters that can be represented by indexes extracted or derived from the market: the VT considering, for istance, the exchange contracts, the incidence of built area and the construction cycle; the VC considering for instance the complementary ratio that represents the incidence of built area (the land under the buildig).

In valuation error analysis of compound values it's possible to define a function of total error that provides a preventive measure of over or under-estimates that can be generated in the compound values because each appraisal can be affected by an error. This function is also a preliminary diagnostic tool and can rationally explain why the international valuation standards recommend caution in estimating the *VT* and not the same is required in the estimation of VC.

#### 2. Valore di trasformazione VT

The VT of a real property that has a capacity to be transformed (eg a building area, a building to be renovated, demolished, or expanded) is equal to the difference between the market value of the transformed property and the transformation cost:

$$S = V - C, \tag{1}$$

where *S* is the *VT*, *V* is the value of the transformed property and *C* is the transformation cost.

The transformation cost is characterized as construction, reconstruction, demolition, preservation, rehabilitation, acquisition cost, etc., depending on the construction, the building and the intervention provided by the same transformation.

The VT can be used to appraise the market value of the building areas, especially when there are not market prices of similar areas recently traded. The VT can be applied to appraise the built area in the case of old or damaged buildings, in the case of additions and plano-volumetric changes (eg added storey) relating to existing buildings, where these measures are technically possible, legal allowed the and cost-effective.

The *VT* of a property that can increase in value by acquisition of another property with a relationship of complementarity, considers the market value of the property to be purchased as a cost.

The VT of a site building or rebuilding ready, or of land into lots, respectively, considers a building or rebuilding process or division into lots, appraising revenues and expenses related to the real estate process (*European Valuation Standards*, *Appendix 1 A 1.87*).

The value of the building area  $V_A$  is in symbolic form:

$$V_A = V - C, \tag{2}$$

where V is the value of the property to be realized and C is equal to the sum of transformation cost and building contractors profit, depending on the building system. Obviously the formula [2] requires a number of clarifications. The construction process is not instantaneous and can be translated into a series of costs and revenues with different payments, and consequently, in financial terms, in the formula the property value represents the present value of revenues and costs, calculated at a given discount rate.

According to the interpretation of the general formula can be distinguished: a practical valuation of building area (and of the *VT* in general) based on the market value and construction cost without considerations on the duration and timing of the construction process, and a valuation based on the discounted cash flow analysis in which the costs and revenues include the sale prices realized and direct and indirect building and promoting costs.

The practical valuation of the building area is applied to rough estimates in the early stages of the construction process and in the valuation reviews.

#### 3. Valore complementare VC

The *VC* of a part of a complex asset is equal to the difference between the market value of the complex asset and the market value of the residual part or parts, once considered separated by the subject. The *VC* is a criterion composed of a combination of two market values: one related to complex asset and the other referred to the residual part or parts.

If  $V_J$  is the VC of the generic part J of a complex property for types, destinations, etc, it could be calculated according to the definition as follows:

$$V_J = V_I - V_{I-j}, \tag{3}$$

where  $V_{l}$ , is the market value of the complex asset and  $V_{l-j}$  is the market value of residual part or parts.

This criterion is also applied in case of partial damage in the expropriation of property, etc. Methodological is used to appraise adjustments in the *market comparison approach*. The relationship of complementarity is essential to identify this valuation criterion whether a part is subtracted from a whole or if a new part is added to an existing whole.

The *ratio of economic complementarity*  $c_J$  is the ratio between the value of the *J* part  $V_J$  and the value of the complex property  $V_T$ , in the following way:

$$c_J = \frac{V_J}{V_J};\tag{4}$$

alternatively this ratio can be calculated considering the residual part or parts of the building, indicating with  $V_{I-I}$  their market value:

$$c_{j} = 1 - \frac{V_{I-j}}{V_{I}}.$$
(5)

## 4. Incidences of built and building area

The incidence of the building area is the ratio between the market value of the building area and the market value of the property to be realized. This market ratio occurs in the exchange contract, where the sale of an undivided share of land, the contract and the division of a building to be constructed, provides the owner of building land to sell an undivided share of the same area to the firm in return of housing units to be built.

The *exchange ratio* r represents the market ratio in a market segment in which the type of contract is the exchange of a building area to real estate units to be constructed and it expresses the relationship between the value of the land  $V_A$  and the market value of the property to be built V:

$$\rho = \frac{V_A}{V};\tag{6}$$

therefore the market value of the building *V* is equal to:

$$V = \frac{V_A}{\rho}.$$
(7)

Substituting the value of the property, calculated using the exchange ratio, in the *VT* formula [2] the value of the building area is reported at cost (M. Ciuna) as follows:

$$V_{A} = V - C = \frac{V_{A}}{\rho} - C = C \cdot \frac{\rho}{1 - \rho}.$$
(8)

The most important ratio of complementarity in real estate is the ratio between the market value of built land and the market value of the property, including the building and the soil. The calculation of the *complementary ratio*  $c_T$  can be developed in two ways: the first will consider the built area value  $V_T$ , obtained from the difference between the market value  $V_A$  and the demolition cost  $C_D$ , and the market value  $V_I$ :

$$c_T = \frac{V_T}{V_I} = \frac{V_A - C_D}{V_I};\tag{9}$$

the second will consider the depreciated reconstruction cost of the building  $C_D$ , obtained from the difference between the cost of new construction  $C_C$  and depreciation D matured at the valuation time, and the property's value:

$$c_T = 1 - \frac{C_D}{V_I} = 1 - \frac{C_C - D}{V_I}.$$
 (10)

The value of the built area by [9] is equal to:

$$V_T = V_I \cdot c_T. \tag{11}$$

The value of the built area is also equal to the *VT* calculated supposing the area free by the building:

$$V_T = V_A - C_D, \tag{12}$$

where  $C_D$  is the total cost of demolition and the demolition profit.

The quantitative relationship that links the incidence of the building area with the incidence of the built area can be found by placing equal the value of the property to build with the value of the built property ( $V = V_l$ ).

According to the formulas [6] and [9], the incidence of the building is calculated as follows from the incidence of the built area:

$$\rho = \frac{V_A}{V} = \frac{V_T + C_D}{V} = \frac{V \cdot c_T + C_D}{V} = c_T + \frac{C_D}{V}.$$
(13)

The inverse formula allows to calculate the incidence of the built area when is known the incidence of the building area, as follows:

$$\mathbf{c}_{\mathrm{T}} = \rho - \frac{C_{\mathrm{D}}}{V}; \tag{14}$$

the ratio between the demolition cost and the property's value shall be interpreted as a penalty in the incidence of the building area respect the built area at a notzero demolition cost.

Sometimes in practical appraisal the complementary ratio of built area takes place in an improper manner by calculating the exchange ratio so that could result an overestimation of the incidence of the built area.

## 5. Valuation error

The absolute valuation error referred to the market price  $E_P$  is the difference between the expected value *S* and the market price *P*, ie:

$$E_p = S - P; \tag{15}$$

the valuation error rate referred to the market price  $e_p$  is equal to:

$$\mathbf{e}_{\mathrm{p}} = \frac{S}{P} - 1; \tag{16}$$

that for values S > P represents an overestimation and for S < P an underestimation. The traditionally accepted maximum threshold of the rate error for the market price valuation is equal to 10% in the monoparametric procedure, this threshold is barely acceptable for the practical valuation or expertise while for scientific valuation based on market data the maximum threshold must not exceed 3 to 5%.

The valuation error rate referred to the construction cost  $e_c$  is measured between the estimated cost *C* and final cost  $C_c$  and is equal to:

$$\mathbf{e}_{\mathrm{C}} = \frac{C}{C_{\mathrm{C}}} - 1; \tag{17}$$

for values  $C > C_C$  represents an overestimation, and for  $C < C_C$  an underestimated.

The expected error in the construction process, for the cost valuation before the project's drawing up is included between -10% and 40%, the intermediate level of 15% is expected for a valuation carried out after the preliminary project and its range is between -5% and 25%, the lower interval is expected for the final and executive project in the range between -3% and 10%.

It is believed that the accuracy of the valuation of the cost is skewed cause of the tendency to protective overestimation in the early stages of the construction process.

#### 6. Valuation error of VT

Appraising the *VT* the valuation error is made by the valuation error of the value and the valuation error of the cost. The final error is a function of partial errors and depends on their signs and their amount.

The *VT* symbolic formula can be expressed by reference to market prices as follows:

$$\mathbf{P} = P_V - C_C,\tag{18}$$

where *P* is the market price of the subject,  $P_V$  is the market price of the transformed property and  $C_C$  is the final cost equal to the sum of the total construction cost, the total promotion cost and of the profit of enterprise (construction and promotion profit).

In practice, the measurement of valuation error covers the market price and the VT, nevertheless it's necessary to measure the partial errors that make the final error. The valuation error rate e referred to the VT is placed equal to:

$$e = \frac{S - P}{P},\tag{19}$$

for S > P is an overestimation and for S < P is an underestimation. The error rate  $e_V$  refers to the *VT* arises equal to:

$$e_{V} = \frac{V - P_{V}}{P_{V}},\tag{20}$$

The error rate refers to the estimated cost  $e_C$  is measured between the estimated cost and the final cost and it is equal to:

$$e_{\rm C} = \frac{C - C_{\rm C}}{C_{\rm C}},\tag{21}$$

for  $C > C_C$  it represents an overestimation, and for  $C < C_C$  an underestimated. The valuation error of cost decreases during the phases of the construction process: by the estimated cost for the preliminary and executive project to the final cost.

From the symbolic formula [18] the valuation error rate of the VT is equal to:

$$\mathbf{e} = \frac{P_V \cdot \mathbf{e}_V - C_C \cdot \mathbf{e}_C}{P};\tag{22}$$

that can be presented as follows:

$$\mathbf{e} = \frac{P_V}{P} \cdot \mathbf{e}_V - \frac{C_C}{P} \cdot \mathbf{e}_C. \tag{23}$$

To study the valuation error of VT is therefore necessary to know the amounts of value of the transformed property and the construction cost. However, it is possible to have a measure of the valuation error studying its applications to building areas appraising. In appraising the building area, the valuation error is applied to the detailed formula of the VT: to the property's value, to the construction cost, to the promotion cost of and profit rates.

Each of these valuations leads to a partial error, which can be computed to the total valuation error by comparing: the estimated value by the market price, the estimated cost with final costs, the expected profit with the profit achieved.

Consider first the formula [6] of the exchange coefficient as follows:

$$\rho = \frac{V_A}{V} = \frac{P_A}{P_V},\tag{24}$$

where the price of the building area is supposed equal to its value. By dividing both sides of the [8] by  $V_{A}$ , there is the following relationship:

$$\frac{C_c}{P_A} = \frac{C}{V_A} = \frac{1}{\rho} - 1;$$
(25)

therefore the valuation error of VT of the building area is equal to:

$$e_{A} = \frac{e_{V}}{\rho} + \left(1 - \frac{1}{\rho}\right) \cdot e_{C}.$$
(26)

The valuation error of the VT of building area is the average of the error rate of the value and of the error rate of the cost, weighted by the inverse of the exchange coefficient. When the exchange coefficient increase, the valuation error rate decreases (*Figures 1* and 2).

The study considers the valuation error in the direction of asymmetric cost overestimates.

For market values the 3% threshold represents the maximum valuation error for scientific appraisal, the 10% represents the maximum threshold for practical valuation or expertise.

For the cost's valuation the 3% is the threshold for scientific valuation, the 10% represents the maximum threshold for practical valuation or expertise (for the cost in the presence of final project), the 15% represents the intermediate level of valuation error in estimating the cost with the preliminary project.



Figure 1. Variation of error with exchange coefficient (asymmetric errors with the same sign %).

*Figure 1* shows the variation of error with the exchange coefficient for asymmetric errors in the cost and with the same sign.

By varying the exchange coefficient r between 0.10 and 0.50, the total error varies:

- between -60% and -4% in case of a 3% overestimation of the value e<sub>v</sub> and a 10% overestimation of the cost e<sub>c</sub>;
- between -105% and -9% in case of a 3% overestimation of the value e<sub>v</sub> and a 15% overestimation of the cost e<sub>C</sub>;
- between -35% and +5% in case of a 10% overestimation of the value e<sub>v</sub> and a 15% overestimation of the cost e<sub>C</sub>.

An overestimation of the value equal in absolute value to an overestimate of the cost, leads to an overestimation of the area equal in absolute value ( $e_v = 10\%$  ed

 $e_{\rm C}=10\%$ ). Other combinations of error produce unacceptable errors, if not for narrow fields of variation of the coefficient of exchange.

The combination of opposite errors produce symmetrical variation respect the x-axis.



Figure 2. Variation of error with exchange coefficient (asymmetric errors with the opposite sign %).

*Figure* 2 shows the variation of error with the exchange coefficient for asymmetric errors in the cost and with the same sign.

By varying the exchange coefficient r between 0.10 and 0.50, the total error varies:

- between -57% and -9% in case of a -3% underestimation of the value e<sub>v</sub> and a 3% overestimation of the cost e<sub>C</sub>;
- between -127% and -23% in case of a -10% underestimation of the value *e<sub>v</sub>* and a 3% overestimation of the cost *e<sub>C</sub>*;
- between -120% and -16% in case of a -3% underestimation of the value e<sub>v</sub> and a 10% overestimation of the cost e<sub>C</sub>;
- between -190% and -30% in case of a -10% underestimation of the value e<sub>v</sub> and a 10% overestimation of the cost e<sub>C</sub>;
- between -165% and -21% in case of a -3% underestimation of the value e<sub>v</sub> and a 15% overestimation of the cost e<sub>C</sub>;

between -235% and -35% in case of a -10% underestimation of the value *e<sub>v</sub>* and a 15% overestimation of the cost *e<sub>c</sub>*.

An underestimation of the value, equal in absolute value to an overestimate of the cost ( $e_V < 0$  and  $e_c > 0$ ), leads an underestimation much larger in absolute value ( $e_v = -3\%$  and  $e_C = 3\%$ ;  $e_v = 10\%$  and  $e_C = 10\%$ ). Other combinations of error estimative produce unacceptable errors.

The combination of opposite errors produce symmetrical variation respect the x-axis.

By varying the exchange coefficient were calculated the valuation reliability as complement of the probability that the error is less than  $\pm 10\%$  considering all combinations of integer values of  $e_V$  between -10% and +10% and  $e_C$  between -5% and +15%, having given the same probability to each combination of  $e_V$  and  $e_C$  errors.

For all combinations of error the valuation reliability stands at very low values ranging from 12.47% to 48.53% for exchange coefficients ranging between 15% and 50%.

For the valuation error with different sign the reliability range is between 1.73% to 24.68% for exchange coefficients ranging between 15% and 50%.

For the valuation error with the same sign the reliability range is between 24.54% to 75.46% for exchange coefficients ranging between 15% and 50%.

The abacus of the errors (*Figure 3*) allows to determine the valuation error  $e_A$  varying  $e_C$  and  $e_V$  at a fixed value r set of example equal to 0.30.



Figure 3. Abacus of errors (r = 30%).

#### 7. Valuation error of VC

Appraising the VC the valuation error is made by the valuation error of the value of the complex property and the valuation error of the property without the subject part. The final error is a function of partial errors and depends on their signs and their amount.

The *VC* symbolic formula can be expressed by reference to market prices as follows:

$$P_{J} = P_{I} - P_{I-J}, (27)$$

where  $P_J$  is the market price of the *J* part,  $P_I$  is the market price of the property and  $P_{I-J}$  is the market price of the remaining portion. In practice, the measurement of valuation error covers the market price of the *J* part, nevertheless it's necessary to measure the partial errors that make the final error. The valuation error rate  $e_J$ referred to the *VC* is placed equal to:

$$\mathbf{e}_{\mathrm{J}} = \frac{V_{\mathrm{J}} - P_{\mathrm{J}}}{P_{\mathrm{J}}},\tag{28}$$

for values of  $V_J > P_J$  is an overestimation and for values  $V_J < P_J$  an underestimate. The valuation error rate  $e_J$  referred to the value of the complex property stands equal to:

$$e_{I} = \frac{V_{I} - P_{I}}{P_{I}}.$$
(29)

The valuation error rate  $e_{l-l}$  related to the residual stands equal to:

$$e_{I-J} = \frac{V_{I-J} - P_{I-J}}{P_{I-J}},$$
(30)

for  $V_{I-J} < P_{I-J}$  represents an overestimate, and for  $V_{I-J} < P_{I-J}$  an underestimate.

From the formula [27] the valuation error rate of VC is equal to:

$$e_{J} = \frac{P_{I} \cdot e_{I} - P_{I-J} \cdot e_{I-J}}{P_{J}};$$
(31)

that can be presented as follows:

$$\mathbf{e}_{\mathrm{J}} = \frac{P_{\mathrm{I}}}{P_{\mathrm{J}}} \cdot \boldsymbol{e}_{\mathrm{I}} - \frac{P_{\mathrm{I}-\mathrm{J}}}{P_{\mathrm{J}}} \cdot \boldsymbol{e}_{\mathrm{I}-\mathrm{J}}.$$
(32)

To study the valuation error of *VC* is therefore necessary to know the amounts of the property value and of the residual value. However, it is possible to have a measure of the valuation error preliminary considering the complementary ratio as follows:

$$c_{\rm T} = \frac{P_J}{P_I},\tag{33}$$

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$$1 - c_{\rm T} = \frac{P_{I-J}}{P_I},\tag{34}$$

consequently the valuation error of the VC is equal to:

$$e_{A} = \frac{e_{I}}{c_{T}} - \left(\frac{1 - c_{T}}{c_{T}}\right) \cdot e_{I-J} = \frac{e_{I}}{c_{T}} + \left(1 - \frac{1}{c_{T}}\right) \cdot e_{I-J}.$$
(35)

The valuation error of the *VC* is the average of the valuation error rate of property value and of the valuation error rate of the value of residual part, weighted by the inverse of the complementary ratio. Increasing the complementary ratio, the valuation error decreases (*Figure 4* and 5).





*Figure 4* shows the variation of error with the complementary ratio for symmetric errors of values and with the same sign.

By varying the complementary ratio  $c_T$  between 0.10 and 0.50 total error varies:

between 15% and -1% in case of a -3% underestimation of the value *e<sub>I</sub>*, and a -5% underestimation of the residual value *e<sub>I-J</sub>*, reaching the 10% error for a complementary ratio equal to 0.14;

- between 60% and 4% in case of a -3% underestimation of the value *e<sub>I</sub>*, and a -10% underestimation of the residual value *e<sub>I-J</sub>*, reaching the 10% error for a complementary ratio equal to 0.35;
- between 40% and 0% in case of a -5% underestimation of the value *e*<sub>*I*</sub>, and a -10% underestimation of the residual value *e*<sub>*I*-*J*</sub>, reaching the 10% error for a complementary ratio equal to 0.25.

An underestimation of the value equal in absolute value to an underestimation of the property without the part, leads to an underestimation of the area equal in absolute value ( $e_I = 3\%$  and  $e_{I-I} = 10\%$ ).

The combination of opposite errors produce symmetrical variation respect the x-axis.



Figure 5. Variation of error with the complementary ratio (asymmetric errors with the opposite sign).

*Figure 5* shows the variation of error with the complementary ratio for symmetric errors of values and with the different sign.

- By varying complementary ratio  $c_T$  between 0.10 and 0.50 total error varies:
- between 195% and 30% in case of a 10% overestimation of the value *e<sub>l</sub>*, and a -10% underestimation of the residual value *e<sub>l-j</sub>*;

- between 95% and 15% in case of a 5% overestimation of the value  $e_I$ , and a -5% underestimation of the residual value  $e_{I-i}$ ;
- between 57% and 9% in case of a 3% overestimation of the value *e<sub>l</sub>*, and a -3% underestimation of the residual value *e<sub>l-l</sub>*.

An overestimation of the value equal in absolute value to an underestimation of the property without the part ( $e_l > 0$  and  $e_{l-l} < 0$ ), leads to an overestimation much larger in absolute value and unacceptable.

The combination of opposite errors produce symmetrical variation respect the x-axis.

## 8. Conclusion

The VT and the VC have the same economic and estimative root. Both are estimated by the difference of two values: a market value and a cost the first and two market values the second, that however can be interpretated, itself to a VT when the value of the remaining part could be view as an annexation cost.

If these values are appraised without any consideration about the cash flow of revenues and costs it allow to formulate hypotheses about the total error made by the partial errors.

The purpose of this paper is to highlight the phenomenon of the effect of the error's combination in the valuation of partial errors and give meaning and content to standards that require caution in applying the compound values. It is not proposed to offer error measures in the expertise, which takes no account of the precautions or standards.

To study the error in appraising the VT is necessary to know the amounts of value of the transformed property and the construction cost. However, it is possible to have a measure of the error studying the VT for the building areas.

The valuation error of the VT of building area is the average of the error rate of the value and of the error rate of the cost, weighted by the inverse of the exchange coefficient. When the exchange coefficient increase, the valuation error rate decreases.

The valuation error of the *VC* is the average of the valuation error rate of property value and of the valuation error rate of the value of residual part, weighted by the inverse of the complementary ratio. Increasing the complementary ratio, the valuation error decreases.

The study of *VT* error varying the coefficient of exchange has highlighted the enormous dimension of the error that is greatly amplified overall in the discordant errors but still remains very high even in the case of concordant errors for which is not obvious the compensation. Only overestimates (underestimates) of the value, equal in absolute value to an overestimates (underestimates) of the cost, leads errors equal in absolute value.

The other combination if discordant are unacceptable if not they are acceptable only for narrow fields of variation of the coefficient of exchange. For all com-

binations of error the valuation reliability stands at very low values ranging from 12.47% to 48.53% for exchange coefficients ranging between 15% and 50%.

Similarly the study of *VC* errors has highlighted the big dimension of the total error, although the thresholds imposed by the scientific valuation of market value based on the real estate data require partial errors lower than those in the cost of the *VT*.

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