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Market Comparison Approach between tradition and innovation. A simplifying approach*

In order to make easy, univocal and accurate Market Comparison Approach application, the present work's aim is to simplify the use of price commercial theorems using numerical coefficients able to correct every single duplication, leading to reliable appraising results and simplified procedures. The work proposes use of a few simple formulas in substitution of the traditional theorems, with the last objective to also standardize procedure as much as possible and also improve results' accuracy.

1. Background

The International Valuation Standards (IVS) indicate as internationally recognized appraising methods the Market Oriented Approach, the Income Approach, the Cost Approach. When real estate market is active and all necessary market data available, Market Comparison Approach (MCA) is the most direct, probative and documented method useful to appraise real estate market values; in particular, Market Comparison Approach (MCA) is the most important method referable to the Market Oriented Approach.

In the Italian real estate context, Market Comparison Approach previews appraising surface characteristics' hedonic prices, using price commercial theorems, reported to the main real estate surface, and some corollaries useful to measure incidence of annexed, connected and external surfaces.

The theorems differ under investigation based on real estate typologies, available data and modalities of price formation in the real estate transactions.

As a general rule, if property does not have external condominium or exclusive surfaces, the analysis turns to the first price commercial theorem, whereby the main surface hedonic price is obtained by the ratio between the sale price and the property's commercial surface. These and other assumptions implied in the numerous theorems and corollaries can however lead to overvalues or undervalue

* Francesca Salvo has drawn the 2nd and the 4th paragraph; Manuela De Ruggiero has drawn the 3rd and the 4th ones. Both have drawn background and shared conclusions.

in many practical cases, with tautologies difficult to interpret and correct, so that their complete and accurate application may be very complex.

The present work's aim is to simplify the use of price commercial theorems using numerical coefficients able to correct every single duplication, leading to reliable appraising results and simplified procedures.

2. Theoretical and practical approximations in price commercial theorems

Market Comparison Approach provides hedonic price analysis using mathematical formulations derived from the appraisal criteria. For surface characteristics, in particular, it uses to refer to the so-called price commercial theorems related to the main real estate surface and some corollaries related to other surfaces. The theorems descend from the real estate transactions practice and are mainly related to the property's typology and to the data's availability, as a function of the total sale price and of the property's commercial surface (Simonotti, 2006).

From a conceptual point of view, however, the main surface hedonic price p_{SUI} can not be referred to the total sale price P but rather to the rate of total price P_{SUI} exclusively referable to the main surface:

$$p_{SUI} = \frac{P_{SUI}}{x_{SUI}} \quad (1)$$

where x_{SUI} is the main surface.

On a practical level, it is not possible to a priori identify the predicted price rate P_{SUI} from the total sale price P , therefore having to resort to an approximation using the real estate sale price P instead of the main surface referred price P_{SUI} :

$$p_{SUI} = \frac{P}{x_{SUI}} \quad (2)$$

In operational terms, the only possibility is to use the total sale price, at the most detracting the impact of the external surfaces as indicated by price commercial theorems (Simonotti, 2006), but without solving the approximation related to the calculation of the main surface hedonic price.

Using the total sale price P , the main surface hedonic price is calculated as:

$$p_{SUI} = \frac{P}{x_{SUI}} = \frac{P_{SUI} + P_{SUB} + P_{SUE} + P_{SER} \dots}{x_{SUI}} \quad (3)$$

in which P_{SUI} is the rate of price due to main surface, P_{SUB} is the rate of price due to balcony surface, P_{SUE} is the rate of price due to external surface, P_{SER} is the rate of price due to the number of restrooms, etc.

The (3) may be also written as:

$$p_{SUI} = \frac{P}{x_{SUI}} = \frac{P_{SUI} + P_{EXT}}{x_{SUI}} \quad (4)$$

where P_{EXT} represents the rate of price due to all the real estate features unless the main surface.

The approximation depends on the presence of the rate of price P_{EXT} , and significantly affects in the adjustments made to the comparable properties' characteristics in the sales adjustment grid, in which each adjustment related to the main surface is calculated as:

$$\left[\frac{P_{SUI} + P_{SUB} + P_{SUE} + P_{SER} \dots}{x_{SUI_j}} \right] (x_{SUI_0} - x_{SUI_j}) \quad (5)$$

and therefore:

$$\left[\frac{P_{SUI}}{x_{SUI_j}} + \frac{P_{SUB}}{x_{SUI_j}} + \frac{P_{SUE}}{x_{SUI_j}} + \frac{P_{SER}}{x_{SUI_j}} \dots \right] (x_{SUI_0} - x_{SUI_j}) \quad (6)$$

where the subscript j indicates the generic comparable, and the subscript 0 the subject.

The exact adjustment to the main surface should be relative just to the rate of total price P_{SUI} :

$$\frac{P_{SUI}}{x_{SUI_j}} (x_{SUI_0} - x_{SUI_j}) \quad (7)$$

whereas in fact it takes into account all other rates of price:

$$\frac{P_{SUI}}{x_{SUI_j}} (x_{SUI_0} - x_{SUI_j}) + \frac{P_{SUB}}{x_{SUI_j}} (x_{SUI_0} - x_{SUI_j}) + \frac{P_{SUE}}{x_{SUI_j}} (x_{SUI_0} - x_{SUI_j}) + \frac{P_{SER}}{x_{SUI_j}} (x_{SUI_0} - x_{SUI_j}) + \dots \quad (8)$$

It may be noted that in the adjustment shown in (8) there are overvalues (and / or undervalues) linked to the presence of price rates different from that closely linked to the main surface.

If in practical terms the only way is to use the total sale price, at least in theory the cited approximations have to be and can be resolved in the subsequent steps, appropriately purifying each one in the adjustment of the corresponding real estate feature.

3. Corrective factors in the sales adjustment grid

The incidence of approximations can be mathematically identified and solved observing the presence of repeated terms in each summand in the general adjustment formula (8). In particular, it is clear that each member in (8) is formed by the product of the single price rate and a term exclusively linked to main surface variables:

$$\left(\frac{x_{SUI_0} - x_{SUI_j}}{x_{SUI_j}} \right) \quad (9)$$

Such term is independent of the specific price rate, depending on the subject's main surface and on the j -th comparable's one, so that it may be written:

$$r_j = \left(\frac{x_{SUI_0} - x_{SUI_i}}{x_{SUI_j}} \right) \quad (10)$$

Using the r_j coefficient the (8) can be written as:

$$P_{SUI} \cdot r_j + P_{SUB} \cdot r_j + P_{SUE} \cdot r_j + P_{SER} \cdot r_j + \dots \quad (11)$$

or, in terms of hedonic prices:

$$p_{SUI_j} \cdot x_{SUI_j} \cdot r_j + p_{SUB_j} \cdot x_{SUB_j} \cdot r_j + p_{SUE_j} \cdot x_{SUE_j} \cdot r_j + p_{SER_j} \cdot x_{SER_j} + \dots \quad (12)$$

In this way, every approximation can be compactly expressed in terms of the r_j coefficient, with the subscript i indicating the generic feature; every overvalue (or undervalue) can be written as:

$$p_{ij} \cdot x_{ij} \cdot r_j \quad (13)$$

where p_{ij} is the hedonic price of every real estate feature different from the main surface.

In light of that previously stated, it can be referred to the first price commercial theorem in any practical case, thereby calculating the hedonic price of the main surface using the total sale price, as shown in equation (2), then to proceed as prescribed by the literature with the determination of the hedonic price of all the other real estate features.

Every possible error (overvalue or undervale) committed in the determination of the hedonic price of the main surface is then removed in the sales adjustment grid.

Each adjustment can then be calculated as:

$$p_{ij}(x_{i0} - x_{ij}) - p_{ij} \cdot x_{ij} \cdot r_j \quad (14)$$

It can be shown that the (14) can also be written as:

$$p_{ij}(x_{i0} - x_{ij}(1 + r_j)) \quad (15)$$

so that the i -th adjustment of the j -th comparable can be written as:

$$p_{ij}(x_{0j} - x_{ij}(1 + r_j)) \quad (16)$$

It can then be generalized the use of the first price commercial theorem, with the foresight to use the minimum of the calculated prices, because this kind of appraisal considers only the drivers characteristics and not all the other real estate features.

4. The adjustment vector

It can be demonstrated that if the subject's characteristics are averages of the surveyed sample, their incidence is nothing in the evaluation, assuming the same value of the *ceteris paribus* features.

In light of this observation, Market Comparison Approach procedure can be significantly simplified by reducing the adjustment table (sales adjustment grid) into a column vector (sales adjustments vector), in which each element is calculated as:

$$\bar{p}_i (x_{i0} - \bar{x}_i) \quad (17)$$

where $\bar{p}_i$ is the average hedonic price of the *i*-th feature, x_{i0} is the *i*-th feature of the subject, while $\bar{x}_i$ is the average of the *i*-th feature in the sample.

The most probable value of the subject is obtained by adding algebraically the elements of the vector, such as:

$$V_{soggetto} = \bar{P} + \sum_{i=1}^n \bar{p}_i \cdot (x_{i0} - \bar{x}_i) \quad (18)$$

Using the correction factor, the (18) becomes:

$$V_{soggetto} = \bar{P} + \sum_{i=1}^n \bar{p}_i \cdot (x_{i0} - \bar{x}_i (1 + \bar{r})) \quad (19)$$

where $\bar{r}$ is the average of the correction coefficients calculated as described by (10).

5. Numerical examples

In order to test the accuracy of the proposed approach, it proceeds to a numerical example based on a concrete appraisal sample concerning flats in condominium, whose features are shown in Table 1.

Table 1. Data Table.

Sale Price and Real Estate Features	A	B	Subject
Sale Price (€)	125.000,00	130.000,00	?
Main Surface (m2) (SUI)	100,00	120,00	106,00
Balcony Surface (m2) (SUB)	10,00	20,000	15,00
External Surface (m2) (SUE)	35,00	15,00	0,00
Restrooms (n°) (SER)	2	2	1
Cellar Surface (m2) (SUC)	15,00	0,00	20,00
Maintenance (point) (MAN)	2	2	1
Floor Level (n°) (LIV)	1	1	3

Basing on data table, every feature's hedonic price is calculated.

The main surface's hedonic price is obtained as the minimum value of the average prices calculated for comparables A and B, as follows:

$$p_{SUI}^A = \frac{P_A}{x_{SUI}^A} = \frac{€125.000,00}{m^2 100,00} = \frac{€}{m^2} 1.250,00 \quad (20)$$

$$p_{SUI}^B = \frac{P_B}{x_{SUI}^B} = \frac{€130.000,00}{m^2 120,00} = \frac{€}{m^2} 1.083,33 \quad (21)$$

The hedonic prices related to balconies and cellar surface is obtained considering their own commercial ratio, because of their commercial relationship, assuming balconies surface ratio in 50% and cellar surface's ratio in 40%, as follows:

$$p_{SUB} = 0,50 \cdot €1.083,33 = \frac{€}{m^2} 541,67 \quad (22)$$

$$p_{SUC} = 0,40 \cdot €1.083,33 = \frac{€}{m^2} 433,33 \quad (23)$$

All the other hedonic prices are calculated as indicated by appraising literature. Table 2 reports the different features' hedonic prices.

Table 2. Hedonic Price Table.

Hedonic Prices	A	B
Main Surface (€/mq)	1250,00	1083,33
Balcony Surface (€/m ²)	541,67	541,67
External Surface (€/m ²)	400,00	400,00
Restrooms (€/n°)	6.000,00	5.000,00
Cellar Surface €/m ²)	433,33	433,33
Maintenance (€/punto)	4.000,00	4.000,00
Floor Level (€/livello)	3.750,00	3.900,00
r_j	0,06	-0,12

The r_j coefficient is calculated using the formula (10):

$$r_A = \left(\frac{x_{SUI_0} - x_{SUI_A}}{x_{SUI_A}} \right) = \left(\frac{m^2 106,00 - m^2 100,00}{m^2 100,00} \right) = 0,06 \quad (24)$$

$$r_B = \left(\frac{x_{SUI_0} - x_{SUI_B}}{x_{SUI_B}} \right) = \left(\frac{m^2 106,00 - m^2 120,00}{m^2 120,00} \right) = -0,12 \quad (25)$$

The r_j coefficients are significant in the sales adjustment grid (Table 3). Just for example, the adjustment for balconies' feature is reported in succession:

$$p_{SUB_A}(SUB_O - SUB_A \cdot (1 + r_A)) = \frac{\epsilon}{m^2} 541,67 \cdot (m^2 15,00 - m^2 10,00 \cdot (1 + 0,06)) = \text{€}2.383,33 \quad (26)$$

In the sales adjustment grid every single adjustment is made explicit; the last record reports the correct prices (Table 3). It's possible to see the convergence of corrected prices, calculated using the corrective factors r_j .

Table 3. Sales Adjustment Grid (€).

Sale Price and Real Estate Features	A	B
Sale Price	125.000,00	130.000,00
Main Surface	(106,00-100,00) x 1083,33 = 6.500,00	(106,00-120,00) x 1083,33 = - 15.166,67
Balcony Surface	541,67x[15,00-10,00x(1+0,06)] = 2.383,33	541,67x[15,00-20,00x(1-0,12)] = -1.444,44
External Surface	400,00x[0,00-35,00x(1+0,06)] = - 14.840,00	400,00x[0,00-15,00x(1-0,12)] = - 5.300,00
Restrooms	6.000,00x[1-2x(1+0,06)] = -6.720,00	5.000,00x[1-2x(1-0,12)] = -3.833,33
Cellar Surface	433,33x[20,00-15,00x(1+0,06)] = 1.776,67	433,33x[20,00-0,00x(1-0,12)] = 8.667,67
Maintenance	4.000,00x[1-2x(1+0,06)] = -4.480,00	4.000,00x[1-2x(1-0,12)] = -3.066,67
Floor Level	3.750,00x[3-1x(1+0,06)] = 7.275,00	3.900,00x[3-1x(1-0,12)] = 8.255,00
Correct Sale Prices	116.895,00	118.110,56
Correct sale Prices (Average)	117.502,78	

Same sample is now used to show the application of the indications discussed in Section 4. Table 4 shows the average of hedonic prices and of correction factors r_j , Table 5 shows the average of sample's features and Table 6 the sales adjustment vector.

The result obtained by applying the simplified procedure converges with the result obtained in the previous application. This convergence is also evident comparing the results obtained through the traditional MCA and those obtained through simplification.

Table 4. Hedonic Prices and r Coefficient (Average).

Main Surface (m ²) (SUI)	1.083,33
Balcony Surface (m ²) (SUB)	541,67
External Surface (m ²) (SUE)	400,00
Restrooms (n°) (SER)	5.500,00
Cellar Surface (m ²) (SUC)	433,33
Maintenance (point) (MAN)	4.000,00
Floor Level (n°) (LIV)	3.825,00
	-0,03

Table 5. Real Estate Features (Average).

Sale Price (€)	127.500,00
Main Surface (m ²) (SUI)	110,00
Balcony Surface (m ²) (SUB)	15,00
External Surface (m ²) (SUE)	25,00
Restrooms (n°) (SER)	2
Cellar Surface (m ²) (SUC)	7,50
Maintenance (point) (MAN)	2
Floor Level (n°) (LIV)	1

Table 6. Adjustment Vector (€).

Sale Price and Real Estate Features	
Sale Price	127.500,00
Main Surface	$(106,00-110,00) \times 1083,33 = -4.333,33$
Balcony Surface	$541,67 \times [15,00-15,00 \times (1-0,03)] = 230,21$
External Surface	$400,00 \times [0,00-25,00 \times (1-0,03)] = -9.716,67$
Restrooms	$5.500,00 \times [1-2 \times (1-0,03)] = -5.188,33$
Cellar Surface	$433,33 \times [20,00-7,50 \times (1-0,03)] = 5.508,75$
Maintenance	$4.000,00 \times [1-2 \times (1-0,03)] = -3.773,33$
Floor Level	$3.825,00 \times [3-1 \times (1-0,03)] = 7.758,37$
Correct sale Prices (Average)	$\Sigma 117.985,67$

6. Conclusions

Market Comparison Approach's traditional formulation, also in the rigorous formal articulation, can lead to overvalues or undervalues in many practical situa-

tions, attended assumptions simplifications connected to the use of the price commercial theorems.

An interesting possibility is the use of corrective coefficients related to each comparable: using corrective coefficients concurs on one side to diminish distortions in the hedonic price analysis, on the other to simplify appraising procedure in operating terms.

The proposed approach generalizes uses of the first price commercial theorem to all appraising situations, sending back comparables' specificities to the adjustments of the various real estate characteristics. The approximation connected to use an only one price commercial theorem, together with other inborn simplifications, resolves in the sales adjustment grid, in which every single adjustment related to the various real estate characteristics is calculated using a numerical coefficient useful to appropriately quantify mentioned imprecision.

Results obtained using corrective coefficients can diverge considerably from those obtained resorting to the classic formulations, in greater measure as well as much more are comparables' complexity they and dissimilarities in real estate sample. The analytic demonstration of the proposed approach, however, leaves to mean the opportunity to resort to the quoted numerical coefficients, whose use is characterized for calculation and operating simplicity.

The application of the Market Comparison Approach can be further simplified by substituting the sales adjustment grid with an adjustment vector, considering mean values in the sample instead of individual comparables' features, comparing the subject with an ideal "average" comparable.

The values obtained using the mean values converge with those obtained by comparing the subject with each comparable, or diverge in minimum measure in presence of features for which mean values lose descriptive significance. However these divergences are negligible in view of the relevant level of simplification of the procedure, greater as much higher is the number of comparables.

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