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# REFERENCE IN FORMAL SEMANTICS AND NATURAL LANGUAGE: A METHODOLOGICAL ROUTE

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## *abstract*

*In this paper, I will tackle the notion of reference of singular terms in the light of a classic analytic divide, i.e. whether its analysis, like the analysis of other basic notions, should be carried out in natural language or in the semantics of formal frameworks. I will incline toward the latter strategy, and consider reference in classical first-order logic as the simplest framework in which to investigate reference.*

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## *keywords*

*reference, arbitrariness, formal languages, natural language, formal semantics*

- 1. Overview** The history of analytic philosophy is scattered with issues related to fundamental notions, which are hard to settle and, at the same time, such that (some) scholars do not seem to agree upon the most sensible methodology to tackle them.
- By *methodology*, I mean the choice between the two horns of a very classic divide in the analytic tradition, i.e. whether in order to tackle those issues it is more reasonable to investigate them within and by *formal languages* or within and by *natural language*. In this respect, there are two radically opposing views: by the first view, one might translate natural language into a formal language, in order to unveil the significance of those basic notions, whose analysis in natural language can be subject to the ambiguities typical of this latter kind of context;<sup>1</sup> by the second, formal languages are supposed to just model notions whose significance and relevance cannot be appreciated but in natural language.
- The aim of this paper is to tackle the notion of *reference* of singular terms, in particular, with respect to whether its analysis, like the analysis of other basic notions, should be carried out in natural language or in the semantics of formal frameworks. I will incline toward the latter strategy, and consider reference in classical first-order logic as the simplest framework we have at disposal.
- In §2, I will provide two classic examples of fundamental notions with respect to the proposed methodological divide. In §3, I will then focus on the notion of reference of singular terms, in order to provide some basis for the main claim that the semantics of formal languages, in particular of classical *first-order logic*, is the right source for the analysis of reference—in particular, supporting a deflationary view of reference, i.e. *arbitrary reference* (§4). In order to carry out this project, a few more issues will be addressed: what is meant by *arbitrariness* and whether arbitrary reference is genuine reference (§5); what semantics for the case study analysed, i.e. classical first-order logic, can be provided (§6). I will then conclude by mentioning some further issues that will need to be tackled in future research and by taking stock of the main claims in this paper (§7).

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<sup>1</sup> This strategy reminds one of Frege's renown scorn for the use of natural language in the investigation of the foundations of mathematics. This attitude brought him to formulate his *Begriffsschrift* in 1879 and, consequently, his colossal and, alas!, fatally flawed *opus magnum*, i.e. the *Grundgesetze der Arithmetik*, 1893-1903.

Let us consider two examples to clarify what is at stake: the notion of *truth*, and the notion of *quantification* in formal languages—in particular with respect to the debate on the ontological commitment of classical second-order logic.

The notion of truth is one of those fundamental notions analytic philosophers have investigated at length, and for the analysis of which different strategies have been envisaged. With respect to the proposed divide, truth may be investigated by recurring to formal tools,<sup>2</sup> regardless of whether this analysis is somehow faithful to the apparent use we make of it in natural language. Consider, for instance, the work of Alfred Tarski, who proved in 1936 the inexpressibility of truth in classical first-order logic. Tarski showed that, as soon as we add a truth-predicate to first-order logic in order to express the truth and falsity of sentences of that language *in that very language*, the so-called *Liar paradox* arises.<sup>3</sup> Tarski's solution to the paradox was to construe a hierarchy of more and more expressive formal languages, each one, apart from the lowest, containing the truth predicate for the language immediately weaker. Tarski's reliance on formal resources implied a rather pessimistic consideration of natural language: natural language is deemed as intrinsically inconsistent, since it is semantically closed, i.e. it contains its own truth-predicate and, thus, reproduces the Liar paradox.<sup>4</sup> With respect to the divide I am proposing, if one agrees with Tarski on natural language, then the correct notion of truth should be the one regimented in a, possibly axiomatic, formal theory of the appropriate form. A second strategy to deal with the Liar paradox might be to revise one or more principles of the underlying logic – whether formal or informal, in order to comply with the manifest functioning of truth in natural language. This latter strategy is the one championed by, e.g., Kripke (1975) and Priest (2006). Though their respective solutions to the Liar paradox are indeed very different logically,<sup>5</sup> they both agree that Tarski's solution provides a rather unnatural reading of the behaviour of the notion of truth in natural language.<sup>6</sup> On the face of the possible strategies on the market, then, it seems reasonable to ask: where does *the* notion of truth come from originally?<sup>7</sup> And by what resources, whether formal or informal, are we supposed to provide an analysis of it?

A further, though somehow restricted, example comes from the philosophy of logic, in particular from the debate on second-order quantification and its ontological commitment. Second-order logic contains two sorts of variables, first- and second-order, and related quantificational resources. The former variables and quantifiers vary over a domain of individuals, the latter vary over a domain of second-order entities – properties, Fregean concepts, sets, classes, or the like. As Quine (1986) famously complained, second-order logic is set-theory in disguise, since its second-order variables are taken to vary over a domain of sets of first-order individuals.<sup>8</sup> To be fair, Quine's claim, in order to be motivated, hinges on two presupposed Quinean slogans, namely

## 2. The Methodological Divide: Truth and Quantification

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2 This approach is underwritten by the many authors who have been working on axiomatic formal theories of truth, from possibly different logical strains, e.g. Hartry Field, Volker Halbach, Andrea Cantini, Leon Horsten, just to mention a few.

3 Famously, the sentence stating its own falsity, which informally might be rendered as e.g. (L) The sentence L is false.

4 And many other paradoxes, for that matter.

5 Kripke (1975) relies on a strong Kleene three-valued logic; Priest (2006), on a three-valued paraconsistent logic.

6 Namely a distinction in infinitely many languages, which, altogether, should capture what truth is. See for instance Kripke (1975, p. 695): “Unfortunately this picture seems unfaithful to the facts. If someone makes such an utterance as (1) [Most (i.e., the majority) of Nixon's assertions about Watergate are false.], he does not attach a subscript, explicit or implicit, to his utterance of 'false', which determines the 'level of language' on which he speaks.”

7 Supposing it would be a great advantage, as I tend to think it would, if there were only one.

8 The correct quotation is “set theory in sheep's clothing”. See Quine (1986, ch. 5).

- (1) to be is to be the value of a variable;
- (2) since intensional entities like properties or Fregean concepts are “creatures of darkness”,<sup>9</sup> whereas sets have a clear identity criterion in the principle of extensionality, sets are to be preferred over intensional entities as the values of second-order variables in modelling second-order logic.

Both presuppositions are questionable to me, still, setting my qualms aside, a received view provides a set-theoretic semantics for second-order logic, by this, committing second-order logic to the existence of mathematical entities. So, for any sentence of natural language that can be expressed in second-order logic, the logical form and ontological commitment of that fragment of natural language are ultimately unveiled by the appropriate formal resources. Boolos (1984, 1985) championed an alternative view on second-order logic, aimed at vindicating its ontological innocence. He provided a semantics in terms of *plural quantification*, whose naturalness and primitivity is justified by examples taken from natural language,<sup>10</sup> (fragments of which are) used as a paradigmatic model for formal languages. To the best of my knowledge, Boolos did not explicitly claim that natural language is indeed the only source of linguistic expressibility we have to take into consideration. Still, on the basis of the debate between Quine and Boolos on second-order logic, it might be tempting to argue that, if both set-theoretic and plural semantics model the language of second-order logic while providing very different models for it, then the modelling of (fragments of) natural language into a formal language is not *per se* indicative of what is expressed and ontologically presupposed by natural language.<sup>11</sup> This latter view is compatible with the claim that, as for the issue of the ontological commitment of quantification in formal languages, natural language can provide models that, at the very least, question views inspired by Quinean assumptions such as (1) and (2) from above.

**3. Reference** The issue I will be interested in, in this paper, concerns yet another fundamental notion, i.e. the notion of *reference*, in particular as for *singular terms* such as proper names or individual constants. My question concerns the nature of this notion, its functioning, and how it comes about, i.e. how it is fixed. My main claim will be that looking into natural language for answers to these questions is not particularly illuminating, since, I claim, the philosophical analysis of reference in natural language is usually carried out by notions that do not come exclusively from semantic considerations and, as such, contribute to blurring it. My main thrust will be that we can go look into formal languages, instead, and devise a form of reference,

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<sup>9</sup> Quine (1956, p. 180).

<sup>10</sup> Especially examples of plural reference in natural language that is not formally reducible to first-order logic, but has to be translated into second-order logic, e.g. the so-called *Geach-Kaplan sentence* “Some critics admire only one another”.

<sup>11</sup> Boolos (1984, p. 449): “The lesson to be drawn from the foregoing reflections on plurals and second-order logic is that neither the use of plurals nor the employment of second-order logic commits us to the existence of extra items beyond those to which we are already committed. We need not construe second-order quantifiers as ranging over anything other than the objects over which our first-order quantifiers range, and, in absence of other reasons for thinking so, we need not think that there are collections of (say) Cheerios, in addition to the Cheerios. Ontological commitment is carried out by our *first-order* quantifiers; a second-order quantifier needn’t be taken to be a kind of first-order quantifier in disguise, having items of a special kind, collections, in its range. It is not as though there were two sorts of things in the world, individuals, and collections of them, which our first- and second-order variables, respectively, range over and which our singular and plural forms, respectively denote. There are, rather, two (at least) different ways of referring to the same things, among which there may well be many, many collections”.

which, unlike reference in natural language, is stripped to its semantic essentials, which is everything we need for reference to work in the first place. So, with respect to the proposed methodological divide, I am definitely on the side of formal languages in this. As Shapiro (1997) stresses,

Probably the most baffling, and intriguing, semantic notion is that of *reference*. (...). How does a term come to denote a particular object? What is the nature of the relationship between a singular term ('Fido') and the object that it denotes (Fido), if it denotes anything? (p. 139).

Again, we have at least two options here: either we go look into the semantics of formal languages or we go look into the semantics of natural language. At first glance, the first route might seem to lead us nowhere:

Notice that model theory, by itself, has virtually nothing to say on this issue. In textbook developments of model theory, reference is taken as an unexplicated *primitive*. It is simply *stipulated* that an 'interpretation' includes a function from the individual constants to the domain of discourse. This is a mere shell of the reference relation. (...) As far as the model-theoretic scheme goes, it does not matter how this reference is to be accomplished or whether it can be accomplished in accordance with some theory or other. (...) As far as model theory goes, reference can be *any* function between the singular terms of language and the ontology (*Ibid.*)

To sum up the quotation from Shapiro, it seems that looking into model-theory for illumination on what reference of singular terms is and how it works is hopeless: any function will do. So, we might as well turn to natural language, in order to see if we can do any better:

If we assume that ordinary languages are understood and if we accept the premise that model theory captures the structure of ordinary interpreted languages, then we can do better. There is, of course, no consensus on how reference to ordinary physical objects is accomplished. The theories are legion. [But at the very least we can presume that] reference to proverbial medium-sized physical objects is accomplished (...) (*Ibid.*, text in square brackets added.)

Nevertheless, even though, on the one hand, the semantics for formal languages might be thought to be of no help in analysing what the notion of reference is and how reference is fixed, on the other, at least two, to some extent incompatible views of reference have been proposed in the analysis of natural language, i.e. Kripke's direct reference view and the classical descriptivist theory of reference supported by Russell and, to some extent, Frege, so that it might be questioned whether the reference relation is captured by any of those views at all.

To this latter claim, it might be objected that, even if natural language is of no more help than the semantics of formal languages with respect to the analysis of reference, at the very least in natural language we have a sense of what seems to be going on.<sup>12</sup> Nevertheless, the attempted

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<sup>12</sup> Notice that reference is taken as a primitive both in the semantics of formal languages and in (most of?) those philosophical pictures in philosophy of language. Reference is not reduced to more basic notions, but scholars have just tried to explain or elucidate it.

analysis of reference and how it is fixed in natural language usually brings influences from different areas of philosophy, not semantics alone, but also e.g. metaphysics or philosophy of mind. In particular, the notions of *individuation* from metaphysics and of *intentionality* from the debate in the philosophy of mind carry a rather crucial import in the analysis of reference in natural language. In this paper, I would like to focus on the impact of the notion of individuation on reference fixing, in particular as for the two classic views on reference in philosophy of language.

A word of caution, though. The notion of individuation may not be particularly transparent conceptually: Lowe (2003) distinguishes between a *metaphysical* interpretation of individuation, and an *epistemic* interpretation of it. By *metaphysical individuation*, Lowe means:

an ontological relationship between entities: what “individuates” an object, in this sense, is whatever it is that makes it the single object that it is – whatever it is that makes it *one* object, distinct from others, and the very object that it is as opposed to any other thing (Lowe, 2003, p. 75).

As Lowe argues, this conception of individuation may be very hard to capture. Epistemic individuation, i.e. the “singling out” of an entity as a “distinct object of perception, thought, or linguistic reference” (Lowe, 2003, p. 75), on the other hand, seems more straightforward, especially since it is often accompanied by some identity criterion.

In this respect, it seems reasonable to point out that the two main opposing views of reference in the philosophy of language seem to require a form of individuation for reference of singular terms to be fixed. Kripke’s initial baptism, in fact, can take place by pointing at an object and tagging it with a proper name – at least when the baptism takes place *in praesentia*, otherwise definite descriptions will do.<sup>13</sup> Also, according to the descriptivist theory of reference,<sup>14</sup> the fixing of reference takes place via definite descriptions, which are a means to single out objects of reference. So, either way, reference via proper names seems to presuppose epistemic individuation in that it requires that objects of reference are somehow singled out, in order for reference of proper names to be fixed. This feature of reference via proper names, shared by the views mentioned above, induces what I like to call *canonical reference*.<sup>15</sup>

#### 4. From Natural Language to Formal Languages

The idea that the fixing of reference of singular terms in natural language is usually accompanied by epistemic individuation is not problematic *per se*. Nevertheless, as soon as we ask for a uniform view of what reference is and how it works, and we take natural language as the only possible starting point in this enquiry, we might also be tempted to claim that the notion of reference in natural language is *the one and only* notion of reference. If, furthermore, we also want to apply this view of reference in natural language to the functioning of reference within formal frameworks, I think it is, first of all, disputable the notion of reference in natural language is to be reliably and exhaustively modelled in formal languages; secondly, implausible that, by extension, it is this kind of reference that takes place in formal languages. Hardly a function of assignment individuates, hardly by description (unless the function is provided with specific instructions) - let alone by ostension. If so, we are forced to assume the existence of at least two different notions of reference, i.e. one in natural language and the other in formal languages. But if we do think that the basic notions analytic philosophy

<sup>13</sup> See Kripke 1980, p. 96 & fn. 42.

<sup>14</sup> E.g. Russell’s and, to some extent, Frege’s.

<sup>15</sup> See also Boccuni & Woods (2018) for similar considerations.

tries to unveil would better be unique, this option seems useless. The alternative view, which is the one I support, is that there is only one basic notion of reference, such that non-semantic considerations are not necessary to fruitfully carry out its analysis. In particular, I do think that we can do only with one, basic notion of reference, which nevertheless does not presuppose individuation, and such that it can be fruitfully analysed in some appropriate semantics for formal languages. In a nutshell, my view is that, though reference is indeed a primitive tool of languages, it can still be stripped down of features that are, in some sense, a philosophical overload, leading thus to a deflated notion of reference i.e. *arbitrary reference*.<sup>16</sup> The outcome is an analysis of reference which is carried out in formal semantics, and such that it can also be reasonably applied to natural language.<sup>17</sup>

In some recent literature,<sup>18</sup> a case has been made for (at least some) singular terms (in mathematical discourse) to be interpreted as *parameters* or *arbitrary names*: terms grammatically behaving like singular terms, but, at least to some extent, semantically functioning as pronouns or indefinites. For instance, in mathematical discourse arbitrary names can be introduced by locutions of the form ‘Let  $n$  be an arbitrary natural number’. This move has the advantage of being proposed on semantic-linguistic (non-metaphysical) grounds, but can be specified in very different, even incompatible, ways. Brandom (1996) provides a survey of the possible options on the table, according to which we may develop two distinct accounts of reference in mathematical discourse.

According to the first account, one may take a liberal view and make a case for two primitive and independent notions of reference: the one I called *canonical reference*; and *arbitrary reference* as embodying some notion of reference other than canonical. This view has the obvious disadvantage of multiplying the notions of reference *præter necessitatem*. Furthermore, this view does not seem appealing, for at least a very general reason: it is hard to justify that we have two different notions of reference, if we have a hard time in capturing what reference should be in the first place. According to the second option presented by Brandom (1996), one may have a confrontational attitude on the issue of reference. This latter view, in turn, may be spelled out in two possible ways:

- (i) (at least some) singular terms are parameters, and they do not refer;
- (ii) singular terms genuinely refer, just not (necessarily) via individuation: what is really basic is arbitrary reference, and, if at all, canonical reference may be built up from it.

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16 For a claim in a similar spirit, though arising from a possible semantics of proper names in natural language, see e.g. Cumming (2008).

17 It might be objected that my view on reference does not imply that the best methodology to tackle this issue requires to analyse it in a formal framework. After all, it would still be compatible with my view that arbitrary reference is a primitive tool of natural language, in which it is already understood, and formal frameworks at best model it, to the effect that the proposed methodological priority of formal over natural language may well be reversed. Nonetheless, unlike in formal frameworks, I find it hard to envisage a clear methodology within natural language to account for arbitrary reference. The only examples of primitive arbitrary reference in natural languages, in fact, seem to be indefinites such as ‘a natural number’. Not all linguists and semanticists agree on the correct interpretation of indefinites, though: indefinites might be treated either as quantified expressions (sometimes existentially, sometimes universally), or as primitively referential, or as essentially predicates with free variables – see, e.g., King & Lewis (2016) and Ludlow (2018) for surveys on these different readings. It seems, then, that the systematic ambiguity of natural language that caused Frege’s scorn is still alive and kicking some a hundred and forty years after the *Begriffsschrift* and regardless the enormous progress in philosophy and linguistics. As things stand, thus, I think it is still methodologically preferable to look into formal frameworks rather than natural language to account for arbitrary reference. I would like to sincerely thank an anonymous reviewer for pointing this out to me.

18 See e.g. Brandom (1996), Pettigrew (2008), Shapiro (2008, 2012), Woods (2014).

Pettigrew (2008) and Shapiro (2008, 2012) seem to pursue option (i), since they take arbitrary names to be non-referential. Applying this view seems uneffective, but I'll get back to it in §5. In what follows, I will pursue option (ii), namely the option by which we still have genuine referentiality, *contra* Shapiro and Pettigrew, and avoid appealing to individuation, thus supporting a metaphysically deflated notion of reference, i.e. *arbitrary reference*.<sup>19</sup>

**5. Arbitrariness and Genuine Reference, i.e. of Love and Logic**

The way in which I like to understand arbitrariness in arbitrary reference might be called the *epistemicist view*. Roughly, this view takes arbitrariness to be epistemic arbitrariness. We cannot *know which* individual an arbitrary term refers to, but the arbitrary term genuinely refers: it refers to a full-fledged individual, equipped with properties. The motivations for this view can be found in natural deduction. Breckenridge & Magidor (2012), Martino (2001, 2004), and Boccuni (2013) conceive of arbitrary reference as genuine reference, and the arbitrariness involved in it as an *epistemic* feature.<sup>20</sup> Mathematicians (and logicians, for that matter) often use locutions like 'Let  $n$  be an arbitrary natural number' in logico-mathematical discourse. These kinds of expressions introduce reference to, e.g., an arbitrary natural number  $n$ , and, according to Breckenridge & Magidor (2012), are governed by the following principle of arbitrary reference:

*Arbitrary Reference:* It is possible to fix the reference of an expression arbitrarily. When we do so, the expression receives its ordinary kind of semantic value, though we do not know and cannot know which value in particular it receives.

According to Martino (2001), the possibility of referring, at least in a non-canonical way, to any entity of a universe of discourse is presupposed both by logical and mathematical reasoning, even when non-denumerable domains are concerned. As a consequence, quantification itself logically presupposes this possibility of referring to each and every element of a domain, before we consider those elements through generalisation. Such a possibility of reference is very well expressed by the crucial role arbitrary reference plays both in formal and informal reasoning. Its cruciality lies in that arbitrary reference exhibits two different logical features that make it essential for performing proofs, i.e. *arbitrariness* and *determinacy*.

Through arbitrary reference, we may consider *any* entity  $a$  of a universe of discourse: as arbitrary reference does not single out specific entities, ' $a$ ' does not refer to an individuated entity  $a$ , rather it refers to an arbitrary  $a$ . Consequently, the epistemic ignorance about which individual  $a$  is, and thus of what its specific properties are, retains the general validity of the arguments about  $a$ . This feature provides a justification for the correct uses of the rule of universal introduction in natural deduction, which sanction as legitimate a deduction from  $\phi a$  to  $\forall x\phi x$  just in case the universal conclusion  $\forall x\phi x$  does *not* depend upon any assumption concerning  $a$ . This restriction is based on the rationale that, in order to legitimately conclude that all  $x$  are  $\phi$  from the fact that an arbitrary  $a$  is  $\phi$ , we have to assure that no specific properties of  $a$  intervene in the deductive step to  $\forall x\phi x$ . Those specific properties of  $a$ , in fact, may not be shared by all the individuals in the domain, and thus the soundness of the deductive step to  $\forall x\phi x$  might be at risk.

At the same time, though, an arbitrary name ' $a$ ' is used to refer to the *same* individual  $a$

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<sup>19</sup> See Boccuni & Woods (2018) for the application of this view to the foundational programme in the philosophy of mathematics known as *Neologicism*.

<sup>20</sup> Of course, there is also Fine's view on arbitrary objects, but taking it into consideration would lead us too far astray, so I shall leave the comparison between my proposal and Fine's to some further work.



within a derivation on  $a$ . Again, a crucial reason for this is to be found in the requirement of soundness we want to impose on some valid argument schemas. If sameness of reference were not a basic ingredient of derivations, soundness would be in jeopardy. Consider the rule of existential elimination in natural deduction. When we pass from a premise of the form  $\exists x\phi x$  to the auxiliary assumption  $\phi(a)$ , ' $a$ ' has to be an unused arbitrary name, or at least it has not to appear in any of the assumptions which  $\exists x\phi x$  depends upon. Consider now the following (invalid) deduction:

- |     |                     |                    |
|-----|---------------------|--------------------|
| (1) | $\exists xHx$       | $\mathcal{A}$      |
| (2) | $\exists x\neg Hx$  | $\mathcal{A}$      |
| (3) | $Ha$                | $\mathcal{A}$      |
| (4) | $\neg Ha$           | $\mathcal{A}$      |
| (5) | $Ha \wedge \neg Ha$ | 3,4 intr. $\wedge$ |

Invalidity stems out from that, in eliminating the existential quantifiers respectively from (1) and (2), we use the very same arbitrary name in (3) and (4). Say that  $H$  is the property of being even and  $x$  varies over the natural numbers: (1) and (2) say, respectively, that there is at least a number which is even and there is at least a number which is not. Both these sentences are true in the standard model of Peano arithmetic. Nevertheless, if we use the same arbitrary name to perform existential elimination in the derivation above, in (3) and (4) we respectively say that a number is even and that *the very same number* is not, from which the contradiction in (5). For this reason, using an already used arbitrary name in (4) cannot be allowed.

In order to explain the invalidity of the derivation (1)-(5) ' $a$ ' must be referring to the *same*, though arbitrary, individual both in lines (3) and (4). Thus, in order to achieve soundness in the previous example, in line (4) we have to use a different arbitrary name than ' $a$ ', because we need to express that a *different* individual than  $a$  is  $\neg H$  within the same derivation, in accordance with the restrictions imposed by the rule of existential elimination. But then again, in order to distinguish between  $a$  and any other arbitrary individual that is  $\neg H$ , we have to assume that  $a$  is a *determinate*, though arbitrary, individual of the domain. The motivation for this requirement is very nicely explained by Suppes:

(...) ambiguous names,<sup>21</sup> like all names, cannot be used indiscriminately. The person who calls a loved one by the name of a *former* loved one is quickly made aware of this. (...) Such a happy-go-lucky naming process is bound to lead to error, just as we could infer a false conclusion from true facts about two individuals named 'Fred Smith' if we did not somehow devise a notational device for distinguishing which Fred Smith was being referred to in any given statement. The restriction which we impose to stop such invalid arguments is to require that when we introduce by existential specification an ambiguous name in a derivation, that name has not previously been used in the derivation (Suppes, 1999, p. 82).

The reasons for restricting the rules of introduction and elimination of quantifiers in natural deduction are semantic: in derivations, we perform a semantic reasoning that we want to proof-theoretically capture by deductive rules and the restrictions on them, in order to retain the soundness of our reasoning. Such a reasoning is crucially based, on the one hand, on arbitrariness and, on the other, on sameness and determinacy of arbitrary reference. But

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21 I.e. arbitrary names.

then again, in order to make sense of sameness and determinacy, and consequently of the requirements we impose on deductive rules for the sake of soundness, we have to assume the genuine referentiality of arbitrary names to begin with. Genuine referentiality is a necessary condition for soundness.<sup>22</sup>

## 6. First-Order Logic and a Semantics for Arbitrariness

Let us now come to a possible semantics for arbitrary reference, and let us consider a formal framework that is as simple as possible: classical *first-order logic* (FOL). There are several semantics that may be utilised to make sense of arbitrary reference in this epistemic nuance, e.g., at least, choice functions, Hilbert-style  $\epsilon$ -terms, and more recently Martino's acts of choice semantics. Regardless what the preferred semantics is, any of them may be used to provide a semantics that fixes the reference of arbitrary names in classical FOL. I will provide a semantics in terms of choice functions, but I think nothing really substantial bears upon it. For the sake of simplicity, let us focus on the monadic fragment  $\mathcal{L}$  of the language of classical FOL, i.e. the fragment containing an infinite list of free variables  $\{x, y, z, \dots\}$ , an infinite list of arbitrary names  $\{a, b, c, \dots\}$ , and an infinite list of monadic predicate letters  $\{F, G, H, \dots\}$ . Let us also assume that the arbitrary names are the only singular terms, i.e. in  $\mathcal{L}$  there are no individual constants.

Let now  $D$  be a non-empty domain, and let  $f$  be a choice function of assignment taking the set of the singular terms of  $\mathcal{L}$  as its domain.<sup>23</sup> In order to model epistemic arbitrary reference, we want  $f$  to assign 'a' to a member of a set of candidate referents from  $D$ . Informally speaking, in order to attach a referent to 'a' we go look into the domain for possible referents, but since 'a' is arbitrary the set of such possible referents may not contain exactly one member: there might be different individuals in  $D$  that are eligible as denotations of an arbitrary name 'a';  $f$  picks exactly one of them, still we don't know which one. So, considering a subset  $C$  of  $D$  containing candidate referents for 'a',  $f$  assigns a member of  $C$  to 'a':

$f: \{a \in \mathcal{L} : a \text{ is an arbitrary name}\} \rightarrow C \subseteq D$ ,

such that:

- (1)  $C \subseteq D := \{i : i \in D \wedge \mathcal{M} \models \Phi\}$ , where  $i$  is an individual in  $D$  and  $\Phi$  is an arbitrary condition expressible in  $\mathcal{L}$ ;
- (2.a) if  $C \neq \emptyset$ , then  $f(a) = i \in C$ , for any  $i \in C$ ;
- (2.b) if  $C = \emptyset$ , then  $f(a) = f(D)$ .<sup>24</sup>

We have to extend this to predicate letters: assuming that predicate letters are interpreted

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<sup>22</sup> Those who support the non-referentiality of arbitrary reference (e.g. Pettigrew, Shapiro) should provide some argument for explaining how classical formal and informal reasoning functions in the way it does, e.g., by certain constraints on introduction and elimination of quantifiers.

<sup>23</sup> Free variables are interpreted as usual, through arbitrary infinite sequences  $S_i$  of individuals of  $D$  that  $S_i$  correlates to free variables, for  $1 \leq i \leq \omega$ .

<sup>24</sup> One might wonder whether, in order to select appropriate candidate denotations for 'a', some form of individuation is still required, since we have to distinguish candidate referents for 'a', which are  $C$ , from those individuals in  $D$  that are  $\neg C$  and, thus, cannot be candidate referents for 'a'. Nevertheless, since the underlying first-order logic is classical and also complete, any subset  $C$  of  $D$  will be such that its complement  $\neg C$  is completely determined, regardless our epistemic ability to single out which objects, if any, are in  $C$  and which are not. Since the logic is classical, we know for a fact that, if any individual at all is in  $C$ , it will not be in  $\neg C$ , and that both these subsets of  $D$  are completely determined with respect to the individuals they contain, even though we might not be able to single those individuals out. I would like to thank an anonymous reviewer for pointing this out to me.

objectually, they are referential as well. Let  $g$  be a choice function of assignment taking the set of all predicate letters of  $\mathcal{L}$  as its domain, and ' $F$ ' a predicate letter. The function  $g$  assigns ' $F$ ' to a member of a class of subsets of  $D$  that are candidate referents for ' $F$ '. Informally speaking, in order to attach a referent to ' $F$ ' we go look into a class of subsets of  $D$  for possible referents, but since ' $F$ ' is interpreted arbitrarily such a class may not contain exactly one member: there might be different subsets of  $D$  that are eligible as denotations of ' $F$ ';  $g$  picks exactly one of them, still we don't know which one. So, being  $\pi(D)$  a class of subsets of  $D$  containing candidate referents for ' $F$ ',  $g$  assigns a member of such class to ' $F$ ':

$$g : \{F \in \mathcal{L} : F \text{ is predicate letter}\} \rightarrow \pi(D),$$

such that

- (3)  $\pi(D) := \{X : X \subseteq D \wedge \mathcal{M} \models \psi X\}$ , where  $X$  is a member of  $\pi(D)$  and  $\psi$  is an arbitrary condition expressible in  $\mathcal{L}$ ;
- (4.a) if  $\pi(D) \neq \emptyset$ , then  $g(F) = C \in \pi(D)$ , for any  $C \in \pi(D)$ ;
- (4.b) if  $\pi(D) = \emptyset$ , then  $g(F) = g(\pi(D))$ .

It might be objected that predicate letters in FOL are constants, functioning as proper names of properties of individuals, and thus my proposed view is not faithful to their intended meaning. Replying this objection is connected to one of the issues mentioned in passing in §4, bullet point (ii), i.e. the possible construal of canonical reference from arbitrary reference. If canonical reference is to be recovered, we can do so by taking arbitrary reference as primitive, and then build canonical reference from it. What is needed is some formal device for expressing individuating conditions, likely in their epistemic nuance i.e. by conditions singling out a specific object of reference. A possible way to do this would be to provide a uniqueness condition by a definite description. The uniqueness condition would impose a constraint on the codomain of a choice function of assignment, i.e. it would work in the limit case where the codomain of the function contains exactly one member. This goes as for both predicate letters such as ' $F$ ' and the introduction of individual constants from arbitrary names.

We can also extend this strategy to the semantics of natural language. All in all, when a Kripkean initial baptism takes place, reference fixing is usually accompanied by some individuating conditions, and this can be modelled as I just suggested. Though, that *de facto* reference fixing in natural language goes with individuation does not imply that the fixing of reference of a proper name and its semantics are necessarily and *a priori* intertwined with metaphysics.

A somewhat more difficult question concerns whether a *general* theory of arbitrary reference can be provided, such that it is applicable to the semantics of any formal theory, whether logical or not. After all, I only provided a semantics for classical first-order logic, and this might raise qualms about the feasibility of extending such arbitrary reference semantics to more complex formal languages. In principle, I see no obstacle to this, since reference is indeed a basic semantic tool of any language, but this topic I shall leave for future research.

In this paper, I investigated one of the most basic notions in the analytic debate, i.e. reference (of singular terms). My aim was to tackle the notion of reference in the light of a classic methodological divide: whether its analysis, like the analysis of other basic notions, should be carried out in natural language or in the semantics of formal frameworks. I argued that the most sensible strategy is to rely on formal resources, because relying on natural language

## 7. Concluding Remarks

implies a philosophically overloaded analysis, weighed down with notions from philosophical areas other than semantics, like e.g. metaphysics. I argued for a deflated notion of reference, namely arbitrary reference, as a genuine, primitive notion, and provided a semantics for it apt for classical first-order logic – the simplest formal framework in which the referentiality of singular terms is relevant.

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